W. A. Costin 3550

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ELECTRICAL COMMUNICATION

Technical Journal of the
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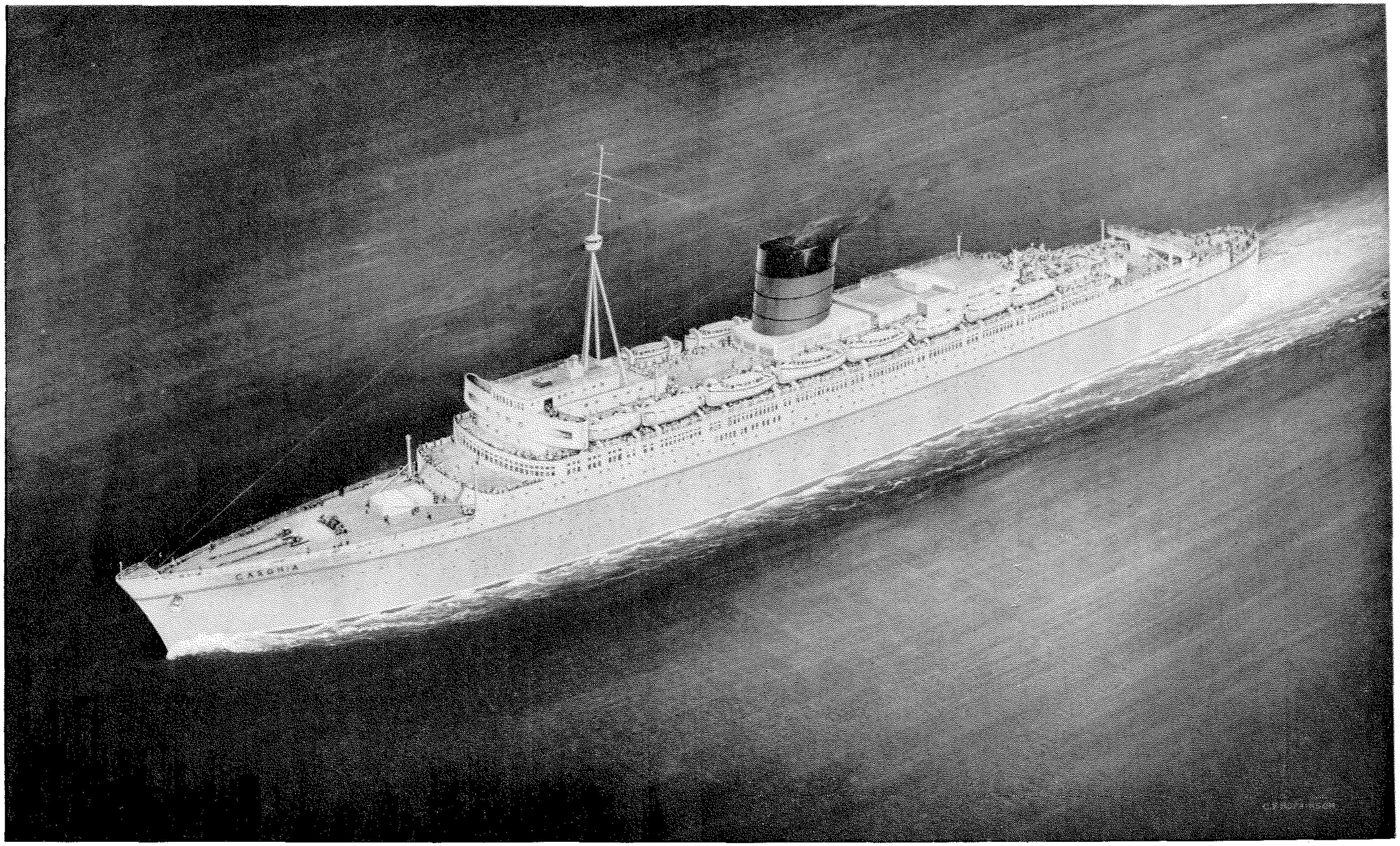


R.M.S. Caronia is the most recent Cunard-White Star liner to be placed in service. As this 34,000-ton vessel must be capable of maintaining direct communication with Europe and America from any place in the world, extensive radio facilities have been provided. Both single-sideband and conventional radiotelephone transmitters and receivers have been installed, making this the first passenger liner in the world to be equipped for single-sideband radiotelephony. Radiotelegraph communication may be handled on high, medium, and low frequencies. The high-frequency radiotelephone equipment was developed and manufactured by Standard Telephones and Cables, Limited, for International Marine Radio Company, Limited, who designed and constructed the radiotelegraph apparatus and were responsible for the entire radio-communication, direction-finding, and public-address installation.

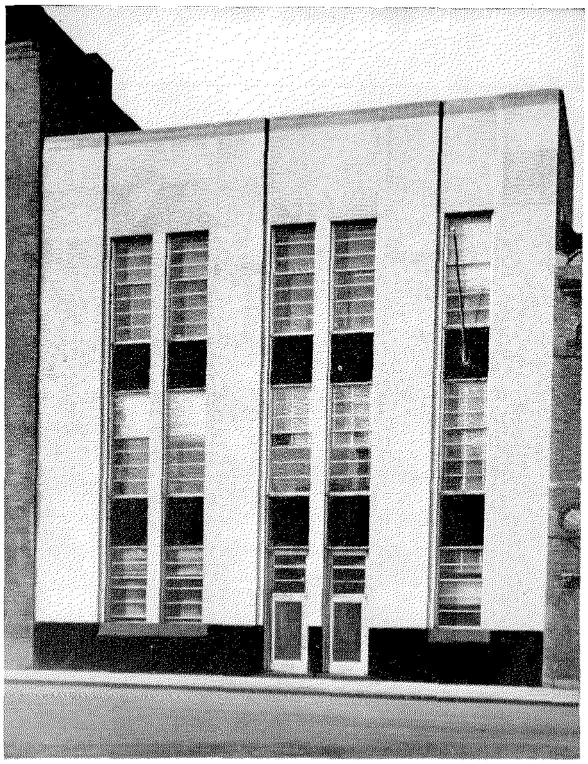
Introduction of Rotary Automatic Service in U.S.A. at Rochester, New York

INSTALLATION of 13,600 lines and 20,000 terminals of 7-A2 Rotary dial switching equipment for the Rochester Telephone Corporation, Rochester, New York, was successfully completed on August 27, 1948. The initial cut-over from manual to dial operation of the Baker and Hamilton exchanges involved 8716 lines and 13,669 terminals.

This is the first installation of 7-A2 Rotary switching equipment in the U.S.A. and is one of the largest cut-overs ever made by an independent telephone company. It is the first of a series of cut-overs that will ultimately convert the entire Rochester network to dial service.

The Rochester Telephone Corporation serves an area of approximately 2300 square miles, including the rich agricultural region within a radius of 50 miles of the city of Rochester, and a population of more than 500,000. With 30 telephones for each 100 persons, Rochester has one of the highest telephone developments in the U.S.A. The Rochester Telephone Corporation is the second largest independent telephone company in the U.S.A. and the city of Rochester with a population of 350,000 is one of the largest served by an independent company.

Long-distance toll service to points outside the operating territory of the Rochester Telephone Corporation is afforded via interconnections with the lines of the New York Telephone Company (Bell System) and its extensive domestic and foreign services. There are 503 toll circuits terminating in Rochester, and on the busiest days more than 34,000 long-distance calls are handled.



New Baker central-office building of the Rochester Telephone Corporation.

Prior to the cut-over of the Baker and Hamilton exchanges, the city was served by a network of eight manual exchanges: Stone, Charlotte, Glenwood, Genesee, Culver, Monroe, Hillside, and Main. The Main exchange building houses the toll offices of both the New York Telephone Company and the Rochester Telephone Corporation. The Stone exchange is in the oldest building, which was built in 1900. The other buildings are of modern construction and house manual exchanges of the latest type designed to give rapid service.

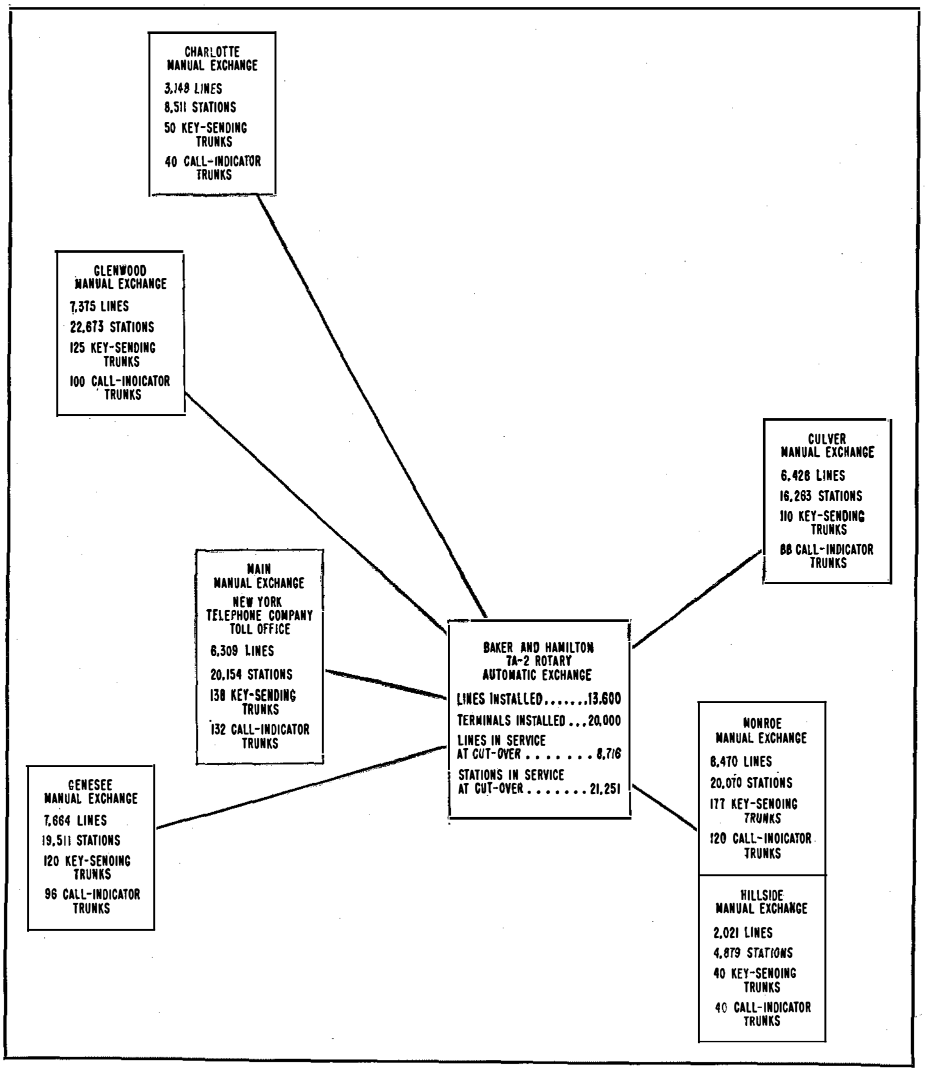
The Stone manual exchange was taken out of service at the cut-over, all lines being transferred to the Baker-Hamilton exchanges, along with part of the lines of the Main exchange.

A new modern three-story Baker central-office building was constructed to accommodate the business office and the rotary equipment for the Baker-Hamilton exchanges. The Baker building will also house the Empire and Locust exchanges now being installed. When completed, the Empire exchange will replace the present pay-station lines and the Locust exchange, the remaining lines of the Main manual office.

The 7-A2 Rotary system[1] was selected with particular regard for its flexibility; it requires a minimum of trunking facilities between offices and allows the change-over from manual to dial operation without disturbing the present manual subscribers' numbering. A further consideration was that the register system incorporated in 7-A2 Rotary switching would meet most adequately the requirements of a multioffice area.

On completion of the installation of the first group of Rotary exchanges at Rochester, the last of which is scheduled for cut-over on May 1, 1949, there will be a total of 18,400 lines and 28,400 terminals. Of these, 13,600 lines and 20,000 terminals are for the Baker-Hamilton exchanges, 1400 lines and 1400 terminals are for the Empire pay-station exchange, and 3400 lines with 7000 terminals for the Locust exchange.

The Rochester equipment consists of 719 bays in the Baker-Hamilton-Empire-Locust automatic exchanges and 194 interworking bays in the manual offices. It was also necessary to convert 269 switchboard positions for direct trunking, and 34 positions for call indicators in the remaining seven manual offices. The installation also includes a 14-position dial-service-assistance board, with two attendants positions for the official private automatic branch exchange; a four-position wire-chief's test panel, a two-position repair-clerk's desk, and a centralized observation desk. The wire-chief's test panels and the repair-clerk's desk are located with the auto-



Rochester network. The Baker-Hamilton office is connected directly to each of the other exchanges by call-indicator trunks for automatic-to-manual interworking and by key-sending trunks for manual-to-automatic interworking.

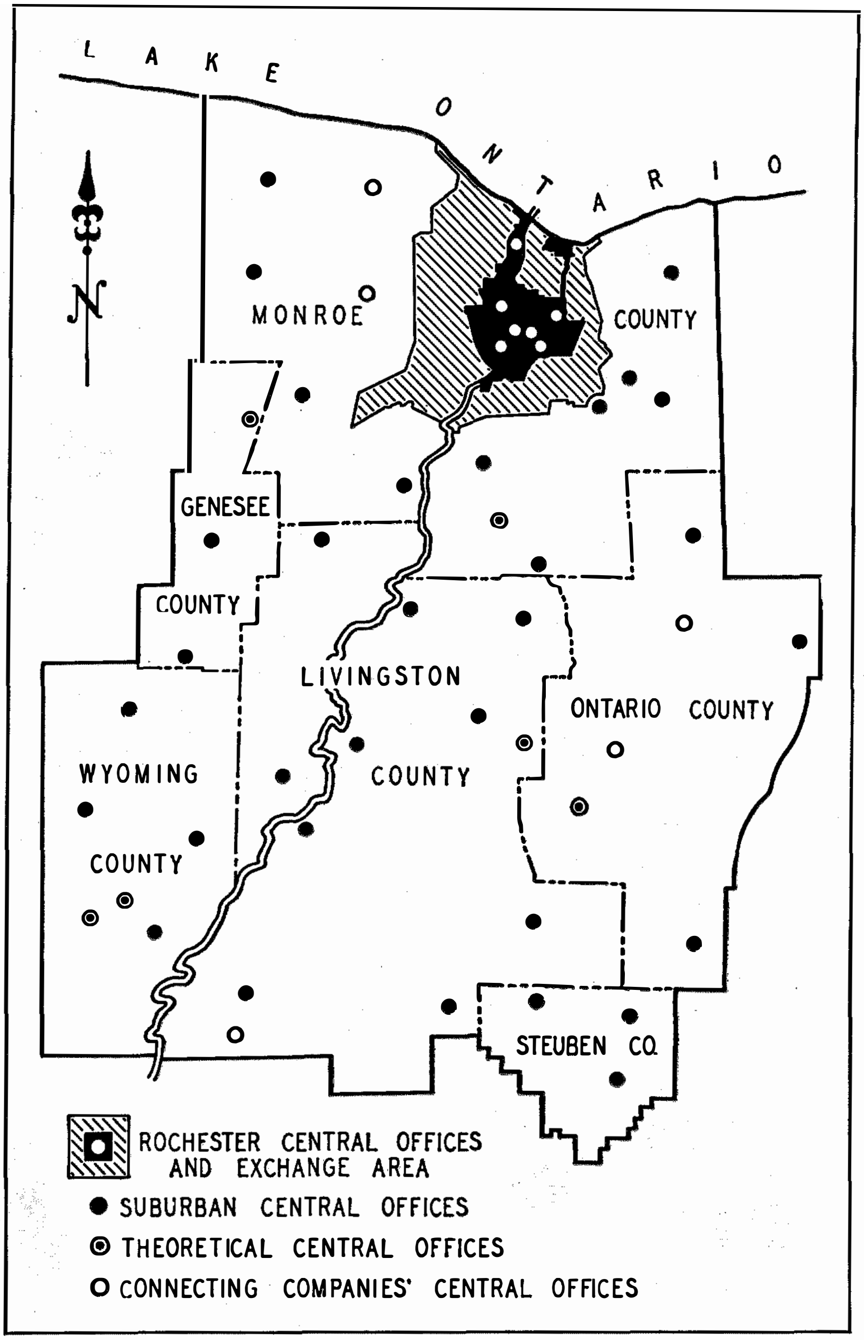
[1] For a description of this system see W. Hatton, "Operating Principles of the 7-A2 Rotary System," *Electrical Communication,* v. 23, pp. 249–264; September, 1946.

matic equipment in the Baker building, while the dial-service-assistance board and other desks are located in the Main exchange building.

Flexible Numbering

The telephone numbering scheme of the Rochester installation permits dial-office subscribers to dial not only other dial-subscribers' numbers, but the complete telephone numbers of subscribers in manual offices. No changes in present manual-office telephone numbers were required. The first two letters of the existing office name is dialed, followed by four numerical digits and party-line letter, if any. The dialing subscriber does not have to pass the number to the operator at any stage of the connection. The operator who completes the connection at the manual office automatically receives the called subscriber's number in a lamp panel known as the call indicator; she merely tests the line for "busy" and plugs in. The proper ringing frequency is selected and applied automatically.

A considerable number of subscribers in the Rochester exchanges are on party lines using harmonic selective ringing. The party-line arrangement is such that any four telephone numbers may be grouped together on a single line. Each party line is equipped with a relay adapter circuit that has four sets of input terminals corresponding to the four final selector terminals associated with the line. The adapter circuit sends a signal to the selector to indicate what kind of ringing current should be applied. One advantage of this flexibility is that regroupings may be made when subscribers move, or for reasons of outside plant efficiency, without changing subscribers' numbers.



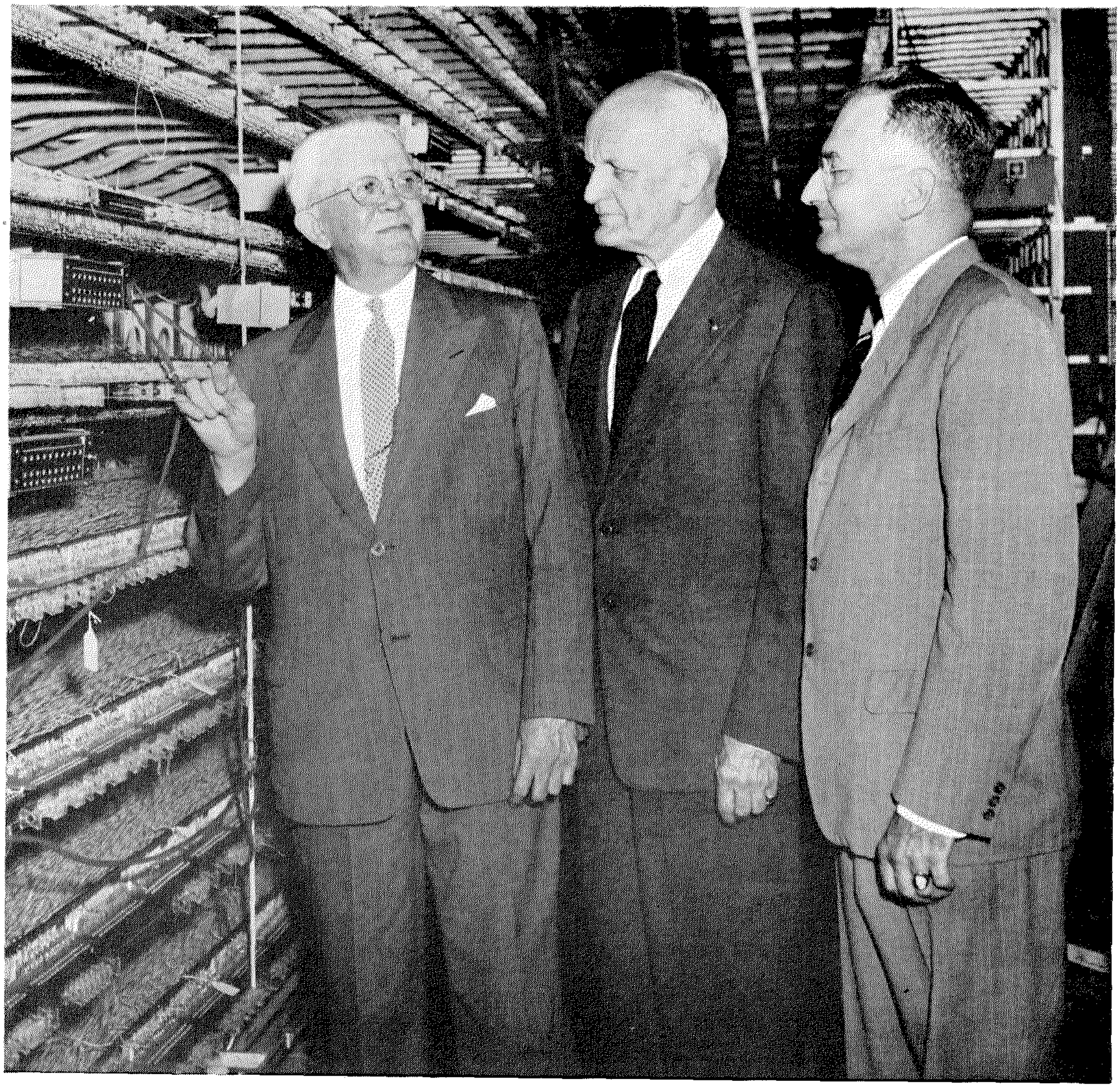
Map of Rochester central offices and exchange area.

Special Features

In the Rotary system, the battery feeds for the calling and called subscriber are in the 1st group selector, except that on interoffice calls the called subscriber battery feed is in the first incoming selection stage. Thus the final selector may be used for toll or local service without modification.

Any final selector may be used as a private-branch-exchange trunk-hunting selector; a special fourth brush is not required for this purpose. The trunk-hunting group may be any group of consecutive numbers served by the final selector —up to the total 200 numbers.

When a call is originated from a pay station in a congested traffic period and no line finders are available, the caller does not lose his coin, as normally happens in such instances. In the case of a pay-station call that exceeds five minutes, a warning tone informs the caller to deposit

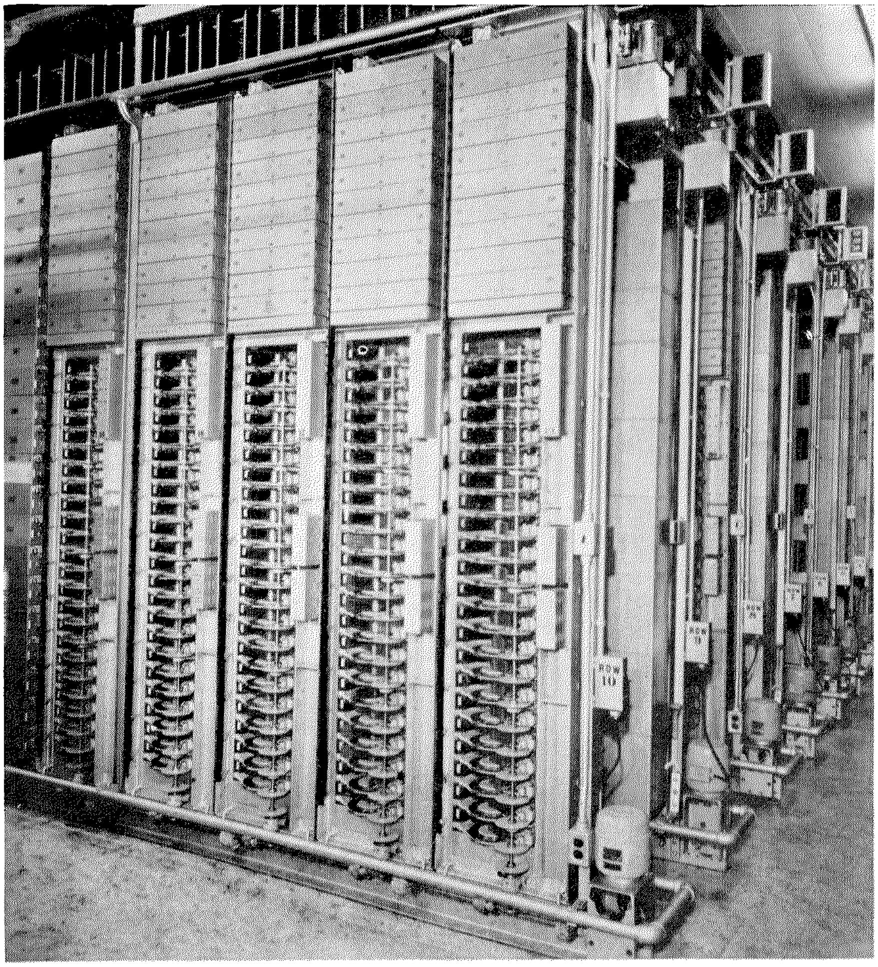


J. W. Morrison, president of the Rochester Telephone Corporation, starts the cut-over while E. W. Stone, president of Federal Telephone and Radio Corporation, and G. W. Miller, chief engineer of the telephone company, look on.

another coin. If the deposit is made, the connection is retained for another five-minute period; otherwise, an operator is automatically called in on the connection to request the additional coin.

The subscriber's message register is on a fourth wire, independent of the cut-off relay, so that the meter and the cut-off relay may each be designed to perform its particular functions best. This arrangement also lends itself to the use of repeated metering when such service is required.

If a final selector stops on a terminal that is not connected to a subscriber's line circuit, a "dead-line" tone is transmitted to the calling subscriber. This is automatically supplied without additional cross-connections by the plant personnel to special dead-line tone circuits or intercept circuits.



First line-finder bays showing drive motors at lower right of each bay.

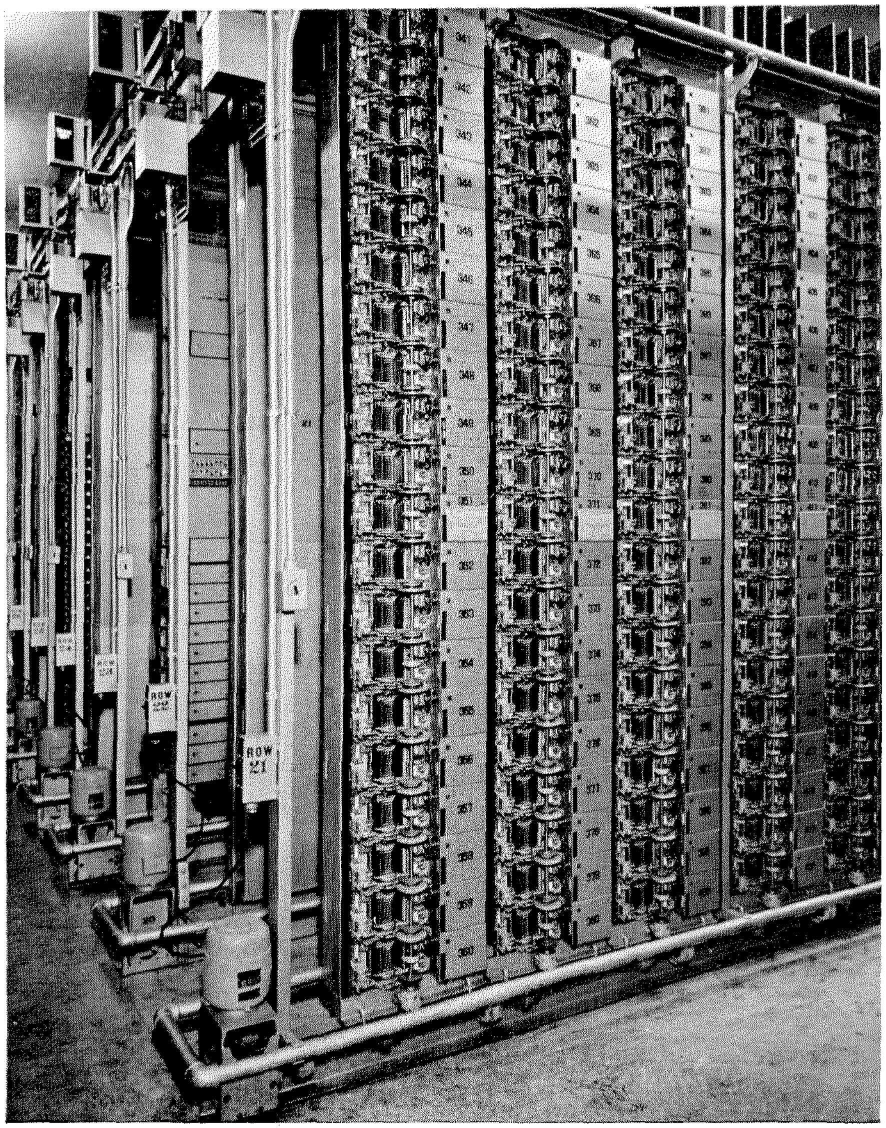
The Rochester installation includes a special toll register designed to operate from 20-impulse-per-second dials in connection with the national toll-line dialing plan of the Bell System and for working with the toll board of the New York Telephone Company. This is the first installation of this type of dialing to be placed in service by an independent telephone company.

A subscriber desiring to initiate a long-distance toll call would dial *110* to reach the toll board of the New York Telephone Company. Short-haul toll calls to tributory points in the system of the Rochester Telephone Corporation are handled through the toll board of that company and the subscriber would dial *0*. Both of these toll boards are located in the Main central-office building.

The 7-A2 equipment is arranged also to work with the Rochester Telephone Corporation's official 7-J private automatic branch exchange, which permits the official subscribers to dial any telephone in the Rochester area.

Maintenance and Testing

The sturdy construction of the Rotary system, made possible by its power drive, together with the system's labor-saving automatic routine testing equipment, hold maintenance personnel and problems to a minimum. With this system, tests are made under most severe operating conditions and corrections often can be made before service is interrupted. A wire-chief's test train is provided and will permit centralized testing of all offices in the multioffice network. The wire chief



A section of the third-group selector bays showing motor mounts.

can obtain a through metallic connection on incoming test calls without having to set up a new connection to the test train.

The necessity to trace down the identity of a line causing a false-call alarm is avoided. When dialing does not proceed within a certain length of time after a call has been originated, the number of the corresponding line is displayed on a lamp panel at the wire-chief's desk.

Acentralized service-observing desk may start observation of a line in a distant office from the instant the line is seized by means of a common selector circuit. The observing operator receives signals showing the identity of the calling line, the number dialed, the coin control and metering signal (if any), and the release by the calling subscriber. The observing operator may release the observing equipment from the line circuit at any time, or she may hold the subscriber's line after he has released.

Built-in permanently wired traffic recording is a feature of the Rotary system. The traffic man may measure the traffic in any important or major group of trunks merely by operating a key. The traffic carried by the trunk group in the succeeding hour will be recorded directly in traffic-intensity units—reflecting the effects of both calling rate and holding time.

Manufacture and Installation

This first installation of Rotary automatic switching in the U.S.A. was made and equipment for it was manufactured by Federal Telephone and Radio Corporation.

(Statistics on "Rotary Automatic Telephone Lines as of December 31, 1948" appear on page 100.)

Fundamental Toll-Switching Plan for Europe of the Comité Consultatif International Téléphonique

By P. E. ERIKSON

International Standard Electric Corporation, London, England

A PREVIOUS ARTICLE in this journal[1] gave a brief sketch of the activities of the Comité Consultatif International Téléphonique (C.C.I.F.), following the cessation of hostilities in Europe. The outstanding feature thereof is a plan known as the "Programme Général d'Interconnexion Téléphonique en Europe (1947–1952)," which is designed on a scale, hitherto not attempted, to provide a rapid international telephone service in Europe. As this project is of considerable economic importance in connection with post-war reconstruction work, a general outline of the plan may be of interest. The general technical requirements from an operating standpoint are described in another article in this journal.[2] The following outline, which is based on the decisions of the Plenary Assembly of the Comité Consultatif International Téléphonique at Montreux, in October, 1946, is confined to the general principles on which the plan is based and to the essential transmission requirements involved.

[1] P. E. Erikson, "Comité Consultatif International Téléphonique," *Electrical Communication*, v. 24, p. 198; June, 1947.

[2] E. P. G. Wright, "Modernization of the International Telephone Service, and Its Reaction on National Telephone Systems," *Electrical Communication*, v. 24, pp. 436–467; December, 1947.

1. General Principles of the Plan

The main purpose of the plan is to provide facilities that will enable two telephone subscribers anywhere in Europe to be connected rapidly over a high-quality circuit.[3] The ultimate goal is for each subscriber to be able to dial another subscriber directly, but the immediate object is to expedite the setting-up of a connection by introducing the single-ticket method, involving only one international operator to record and supervise the call.

As a fundamental principle, the Comité Consultatif International Téléphonique recommends that a typical international telephone connection with manual operation should not contain more than two international circuits in tandem, and each national system not more than two toll circuits in each system. This set-up is illustrated in Fig. 1, from which it will be seen that only one international transit centre is involved in the connection. Considering the overall circuit from the calling subscriber's end (national

[3] It is to be understood, however, that when the traffic between any two centres warrants a strong group of circuits, such calls shall be routed over direct circuits and not be switched.

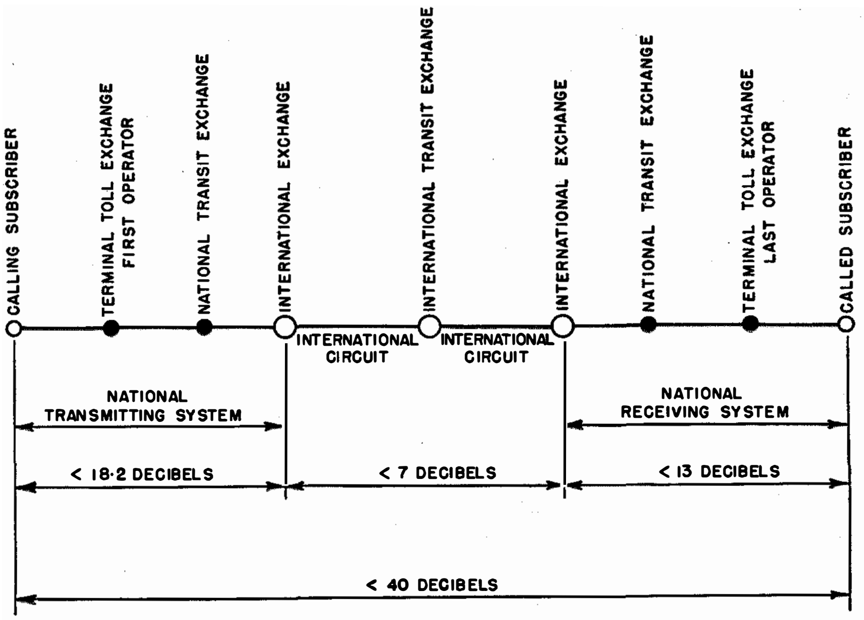


Fig. 1—Typical international telephone connection in Europe.

transmitting system), it is the operator in the toll exchange, who controls the call and makes out the only ticket used as a record of the call.

Normally a national transit exchange has direct circuits to all its dependent toll offices in the country and to at least one international exchange. The function of the international transit exchange is to establish connections between two countries (other than its own) and it should therefore have direct circuits to the appropriate exchanges in those countries. Each international transit exchange should have connection facilities to all the others, either by direct circuits or through the intermediary of other international transit exchanges.

For the routing of international calls in the European service, the Comité Consultatif International Téléphonique recommendations cover three exigencies for directing the traffic over:

A. Normal routes.

B. Auxiliary routes, which are used whenever this expedient promotes a rapid service.

C. Emergency routes, used in case of complete breakdown of the normal and auxiliary routes.

The number of circuits connecting two toll exchanges should be sufficiently large to handle the traffic on a good-quality-service basis. The quality is provisionally defined as the percentage of connections that, during the average busy hour, cannot immediately be completed, due to lack of circuits. A connection is said to be handled immediately if the operator is able to complete it within two minutes of the receipt of the call, even if she fails to find a free circuit immediately and continues to search over the group of circuits or makes several attempts during this period. The number of circuits required to give a good-quality service is a function of the total holding time on the group during the busy hour. The total holding time is the number of calls during the busy hour multiplied by a factor that is the sum of the average conversation time and the average operating time.

The total holding time of a circuit during the busy hour having been determined, a certain allowance is made for future increase in traffic, based on past experience. It is then possible to compute the total holding time for a group of circuits. The number of circuits per group is related to the percentage of total circuit time occupied and the possible occupied minutes in the busy hour in the manner shown in Table 1.

Scale *A* applies, in general, to the present condition of the international system. According to theoretical studies, based on experimental data gained from rapid operation of their national systems, various administrations have reached the conclusion that the circuit capacities indicated correspond to about 30 percent of calls ineffective at the first attempt due to all circuits being engaged and to 20 percent of calls cancelled for other reasons.

Scale *B*, which is based on a somewhat more rapid service, is recommended whenever carrier or coaxial cables are used in the international service because of the considerable increase of circuits that is economically obtained by their use. It is estimated that by adopting scale *B* the number of cancelled calls will be reduced to about 7 percent.

No account has been taken in these tables of the fact that alternative routing of the traffic may permit an increase in admissible holding time, particularly in the case of very small groups.

TABLE 1

SCHEDULE OF GROUP-CIRCUIT CAPACITIES

Number of Circuits	Scale A		Scale B	
	Percentage of Total Circuit Time Occupied*	Possible Occupied Minutes in the Busy Hour	Percentage of Total Circuit Time Occupied*	Possible Occupied Minutes in the Busy Hour
1	65.0	39	—	—
2	76.7	92	46.6	56
3	83.3	150	56.7	102
4	86.7	208	63.3	152
5	88.6	266	68.3	205
6	90.0	324	72.0	259
7	91.0	382	74.5	313
8	91.7	440	76.5	367
9	92.2	498	78.0	421
10	92.6	556	79.2	475
11	93.0	614	80.1	529
12	93.4	672	81.0	583
13	93.6	730	81.7	637
14	93.9	788	82.3	691
15	94.1	846	82.8	745
16	94.2	904	83.2	799
17	94.3	962	83.6	853
18	94.4	1020	83.9	907
19	94.5	1078	84.2	961
20	94.6	1136	84.6	1015

* C.C.I.F. Recommendation No. 1, Par. 18 (Traffic).

Such routes will, in fact, rarely be used in the international service.

It is evident that a fast international service can only be achieved by means of a network specially designed for that purpose and, in a sense, divorced from the national network. It is equally clear that such an international plan must provide large circuit groups between transit centres to promote a rapid flow of traffic. The European international service depended in the past on the utilization of national circuits, supplemented by many long and numerically weak direct circuits between countries. In other words, the international network of old had never been designed as an entity, but was to a great extent superimposed on existing facilities.

An analysis of the groups of international circuits that existed in 1939 has been made, and the conclusions reached are of interest as showing what happens under such conditions. Briefly stated, it was found that 49 percent of the groups contained only one circuit and that 90 percent of all groups were to be found in groups containing 1 to 4 circuits. The average group contained 2.5 circuits. Also, that 20 percent of all the circuits are found in the one-circuit group, whilst 60 percent fall into the 1-to-4-circuit groups. Using scale *A* of Table 1 as a measure of service, it was ascertained that only 16 percent of all the available busy-hour minutes could be obtained from the one-circuit group, whereas 55 percent derived from the 1-to-4-circuit groups. These contrasts are brought out in Table 2.

TABLE 2

Item	1-Circuit Groups in Percent	1-to-4-Circuit Groups in Percent
Groups	49	90
Circuits	20	60
Minutes	16	55

As a matter of interest, Table 3 has been compiled to enable comparisons to be made between the number of international circuits that existed in 1939 with those estimated as required in 1952. It will be noted that, in the case of some of the larger circuit groups, the ratio of the estimated (1952) circuits to those of 1939 is in the order of 4:1. For the relatively smaller circuit groups the same ratio is considerably higher.

2. *Application of the Plan*

The plan as approved by the Plenary Assembly at Montreux in October, 1946, is shown on the map (Fig. 2).

It is assumed that where large groups of direct circuits between the capitals of contiguous countries exist, such groups will remain unchanged. In some instances, these circuits would be assigned to terminating traffic because of the inherent characteristics of the circuits. In case such circuits are designed to conform with modern technique, or can be modified along those lines, they may be suitable for inter-country traffic without any restrictions and may even be used for transit traffic.

The map shows the location of the international transit centres, in addition to which there are national transit centres at suitable distribution points. Cables, preferably of the coaxial type, with ample capacity for augmenting circuits when required, would connect together the transit centres. Each country is expected to provide an adequate number of traffic channels from its international terminal exchange to one or more of the transit centres, through which the long-distance international traffic would flow.

The plan tends to eliminate independent small traffic groups of the kind discussed above, but some medium-sized groups would probably be retained where traffic conditions may justify their use.

Signalling and switching arrangements for transit centres would be designed in accordance with a general specification, laying down fundamental performance requirements but not restricting the apparatus to be used to any particular type. It is not expected that the plant in the proposed plan will be of a rigidly uniform design. There are admittedly both advantages and disadvantages in rigid standardization and it is important to discriminate between essential and non-essential requirements. In some respects, particularly as regards overall performance, it will be necessary to conform to a common international specification; in other cases, such as, for example, general equipment design, national preference would prevail

It is not necessary, nor would it be feasible, to bring the whole plan into operation at one time. An important consideration is, however, that the design of the cables and equipment be such as to make future extension economically possible. For example, a coaxial cable, capable of yielding hundreds of speech channels, can be installed, once and for all, together with its repeater equipment. Depending on the initial traffic requirements, sufficient terminal equipment is provided and, later on, added to as the traffic grows. The first cost of the transmission system thus comprises cable, repeaters, and sufficient terminal equipment to meet immediate traffic demands, whilst the cost of additional terminals is deferred.

Economic studies, which have been made by various authorities, show that the coaxial-cable system is by far the cheapest method of providing underground circuits for international use both with regard to first costs and present values of annual charges. As an illustration, it may be mentioned that for equal circuit facilities a comparison between a coaxial-cable system and a system using 24-channel carrier equipment on a paper-insulated cable shows that the ratio between the present values of the annual charges is 0.78 in favour of the coaxial system. In this case, the period involved in the study was 15 years, during which the traffic handled by either system was assumed to grow at the same rate. In Great Britain, it has been found that the →

TABLE 3

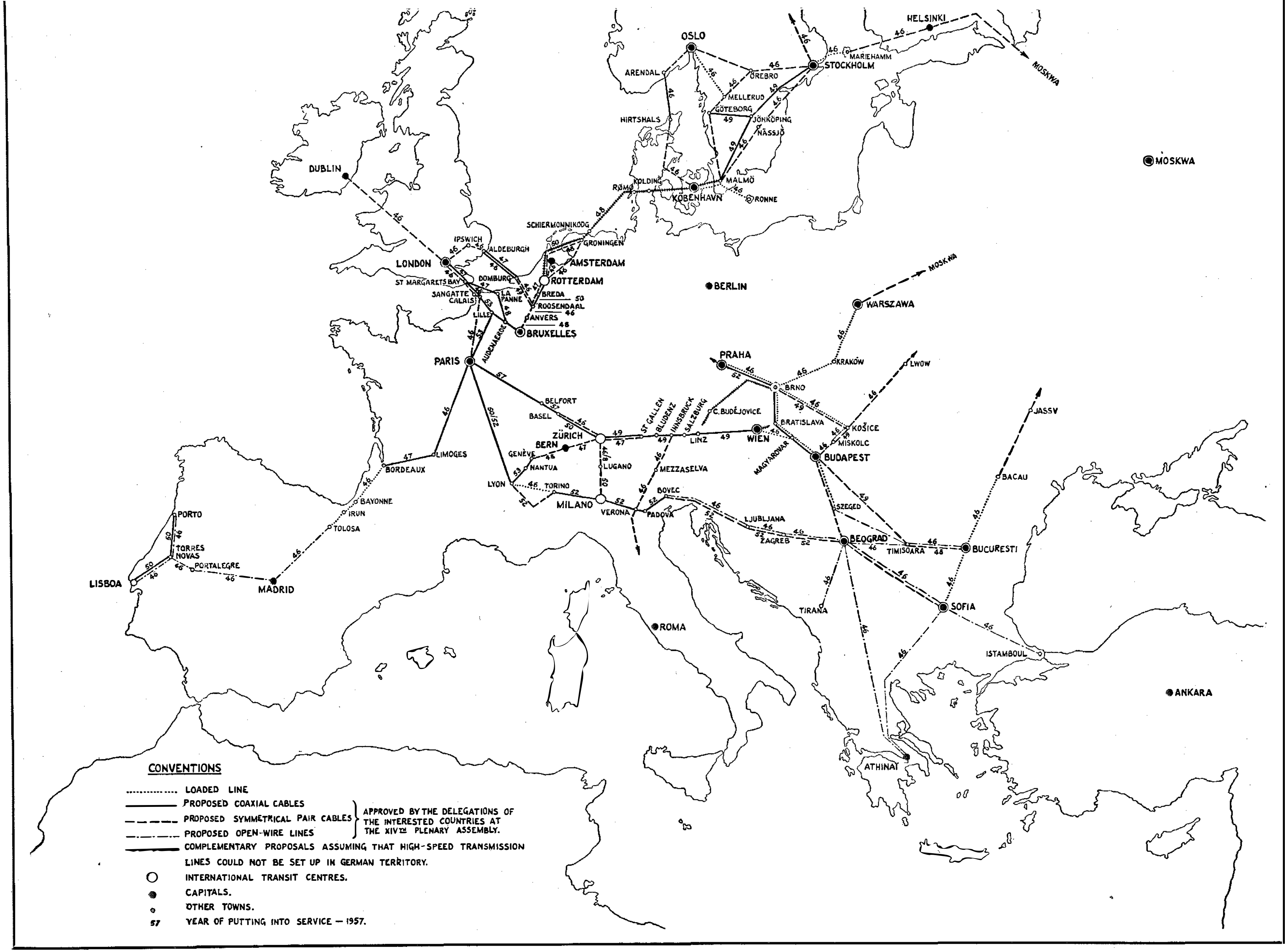
EUROPEAN INTERNATIONAL CIRCUITS

Figures **Above** Diagonal Line of Asterisks Are the Number of Circuits in 1939, and Those **Below** Are the Number of Circuits Estimated as Necessary in 1952.

	Austria	Belgium	Czechoslovakia	Denmark	Finland	France	Great Britain	Hungary	Italy	Netherlands	Norway	Poland	Portugal	Roumania	Spain	Sweden	Switzerland
Austria	*	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Belgium	12	*	1	1	0	79	22	1	2	38	0	0	0	0	0	1	7
Czechoslovakia	24	12	*	1	0	3	2	9	2	3	0	5	0	3	0	0	3
Denmark	0	12	12	*	1	1	4	0	0	2	5	1	0	0	0	18	1
Finland	0	0	0	12	*	0	0	0	0	0	0	0	0	0	0	13	0
France	24	300	24	12	0	*	66	2	9	19	0	3	0	2	10	2	49
Great Britain	12	84	12	24	0	228	*	2	6	27	3	2	0	1	0	5	14
Hungary	24	12	24	0	0	12	12	*	4	1	0	2	0	9	0	0	4
Italy	12	12	12	0	0	72	48	12	*	2	0	1	0	0	0	0	43
Netherlands	12	144	12	12	0	60	120	12	12	*	0	1	0	0	0	2	13
Norway	0	0	12	48	0	12	24	0	0	12	*	0	0	0	0	9	0
Poland	24	12	24	12	0	24	12	12	12	12	12	*	0	1	0	1	2
Portugal	0	0	0	0	0	12	12	0	0	0	0	0	*	0	4	0	0
Roumania	12	0	24	0	0	12	12	24	12	0	0	12	0	*	0	0	0
Spain	0	0	0	0	0	24	12	0	0	0	0	0	24	0	*	0	2
Sweden	12	12	12	60	36	12	36	0	0	12	60	12	0	12	0	*	1
Switzerland	60	36	24	12	0	120	84	24	72	36	12	12	12	12	12	12	*

Note: In 1939, Austria formed part of Germany.

Fig. 2—Map of European network of high-speed transmission lines as part of the Comité Consultatif International Téléphonique general telephone switching plan for Europe, 1947–1952.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HIRTSHALS
DUBLIN
MALMÖ
KOLDING
KØBENHAVN
RØNNE
SCHIERMONNIKOOG
GRONINGEN
IPSWICH
ALDEBURGH
AMSTERDAM
LONDON
DOMBURG
ROTTERDAM
ST MARGARETS BAY
SANGATTE
CALAIS
LA PANNE
BREDA
ROOSENDAAL
ANVERS
LILLE
BRUXELLES
BERLIN
WARSZAWA
PRAHA
KRAKÓW
LWOW
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PROPOSED COAXIAL CABLES
PROPOSED SYMMETRICAL PAIR CABLES
PROPOSED OPEN-WIRE LINES
APPROVED BY THE DELEGATIONS OF THE INTERESTED COUNTRIES AT THE XIVth PLENARY ASSEMBLY.
COMPLEMENTARY PROPOSALS ASSUMING THAT HIGH-SPEED TRANSMISSION LINES COULD NOT BE SET UP IN GERMAN TERRITORY.
INTERNATIONAL TRANSIT CENTRES.
CAPITALS.
OTHER TOWNS.
57 YEAR OF PUTTING INTO SERVICE — 1957.

coaxial cable system is economical for distances as short as 70 kilometres (44 miles).

It should be pointed out, however, that the European toll-switching plan is not based solely on the use of coaxial cables but rather on the use of circuits with high speed of propagation. In cases where only a small number of circuits is required (and no appreciable growth is expected), it may be more economical to use open-wire carrier circuits.

3. *Summary of Transmission Requirements*

The Comité Consultatif International Téléphonique has accepted the principle of evaluating transmission quality in terms of "effective rating," which was introduced as far back as in 1931. However, the practical methods of applying this principle have not yet been worked out to the satisfaction of the European administrations and in the meantime transmission planning is based on reference equivalents as heretofore but taking into consideration the impairments due to line noise and to limitation of frequencies transmitted by the national trunk lines.

4. *Overall Reference Equivalents*

The proposals amount in effect to setting an absolute limit to the reference equivalent of an international connection equal to 40 decibels, less the impairments in the national networks due to noise and reduction of frequency transmitted.

The absolute limit thus determined includes temporal variations in the national and international networks and an allowance for the minimum acceptance limits of transmission efficiency specified for the telephone sets.

In the case of no impairments attributable to noise and cut off in the national networks, the breakdown of reference equivalents should be as follows.

The reference equivalent of the national transmitting system should not exceed 18·2 decibels and that of the national receiving system should not exceed 13 decibels. The net loss of the international connection measured at the switchboard

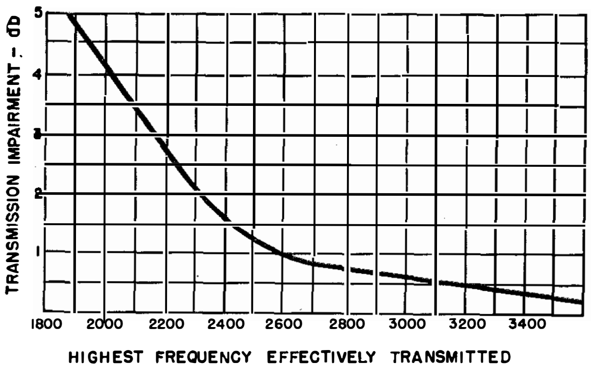

Fig. 3—Frequency effectively transmitted when the net loss is not more than 10 decibels above the net loss at *1000* cycles per second.

jacks, between 600-ohm terminations, has been standardised at 7 decibels at 800 cycles per second. This figure includes the cord circuit at the transit switching exchange when the international connection consists of two international circuits in tandem. The variation of the net loss with time should not exceed 1·7 decibels.

5. *Correction for Reduction of Transmission Quality Due to Noise and to Limitation of Band Width Effectively Transmitted*

Measurements made some years ago in the S.F.E.R.T.[4] laboratory have been accepted as a

[4] Système Fundamental Européen de Référence pour la Transmission Téléphonique (Comité Consultatif International Téléphonique laboratory at Geneva, Switzerland).

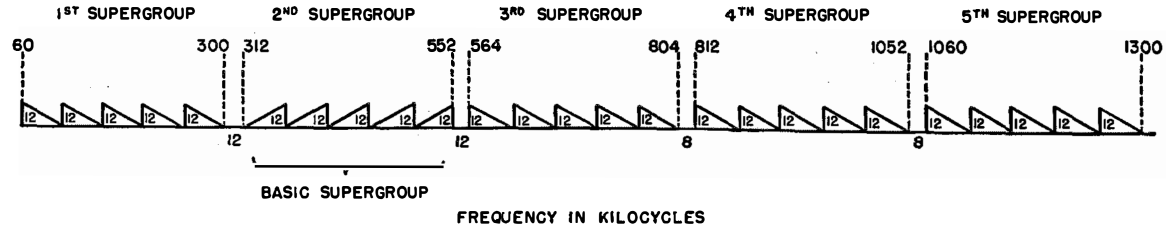

Fig. 4—Distribution of line frequencies in coaxial systems. Each triangle represents a basic group of 12 channels with virtual carrier frequencies spaced by 4 kilocycles.

basis for relating the noise on an international connection to transmission impairment. Similarly, limitation of band width effectively transmitted has also been related to transmission impairment. Transmission impairment is expressed in decibels and the sum of the two impairments is added to the nominal net loss of the international connection at 800 cycles.

TABLE 4

Psophometric Electromotive Force in Millivolts	Transmission Impairment Due to Circuit Noise in Decibels
<2.5	0
2.5 to 4.0	1
4.0 to 5.5	2
5.5 to 7.0	3
7.0 to 8.5	4
>8.5	5

Table 4 compares weighted noise electromotive force and transmission impairment. Fig. 3 is a curve showing the relation between the highest frequency effectively transmitted and transmission impairment in the presence of average room noise.

It was also recommended that the effects of stability and echo should be considered and methods for calculating these effects have been standardised.

The transmission recommendations apply to all types of circuits used for international service. Studies, which have been proceeding since the 13th Plenary Assembly (London, 1945), have been directed towards formulating recommendations applicable to high-speed high-quality circuits such as will form the basis of the European general telephone switching programme. The following recommendations were approved by the 14th Plenary Assembly (Montreux, 1946) or were provisionally adopted by the 3rd C.R.[5] at the preliminary meeting in Paris in May, 1947.

A. The spacing between virtual carrier frequencies in all types of carrier systems having 12 or more channels should be 4000 cycles.

B. The frequencies transmitted to line should be as follows.

a. *12-channel carrier-on-cable systems:* 12 channels in the range 12 to 60 kilocycles. The transmitted side-band of each channel should be the upper side-band of the virtual carrier frequencies, 12, 16 ... 56 kilocycles. This group of side-bands is termed group *A*.

b. *24-channel carrier-on-cable systems:* 24 channels in the range 12 to 108 kilocycles composed of two groups of 12 channels, namely, group *A* as above and group *B* consisting of the lower side-bands of the virtual carrier frequencies 64, 68 ... 108 kilocycles.

c. *Coaxial systems:* The line frequency distribution should conform to Fig. 4. It is made up of primary groups of 12 channels each and secondary groups, or supergroups, each of 5 groups (60 channels).

C. *Band Width Effectively Transmitted:* All high-speed international circuits should transmit effectively the band from 300 to 3400 cycles, which corresponds to a 4000-cycle spacing between virtual carrier frequencies. A frequency is said to be effectively transmitted when the net loss at that

[5] 3rd Commission de Rapporteurs of the Comité Consultatif International Téléphonique, charged with the study of transmission questions.

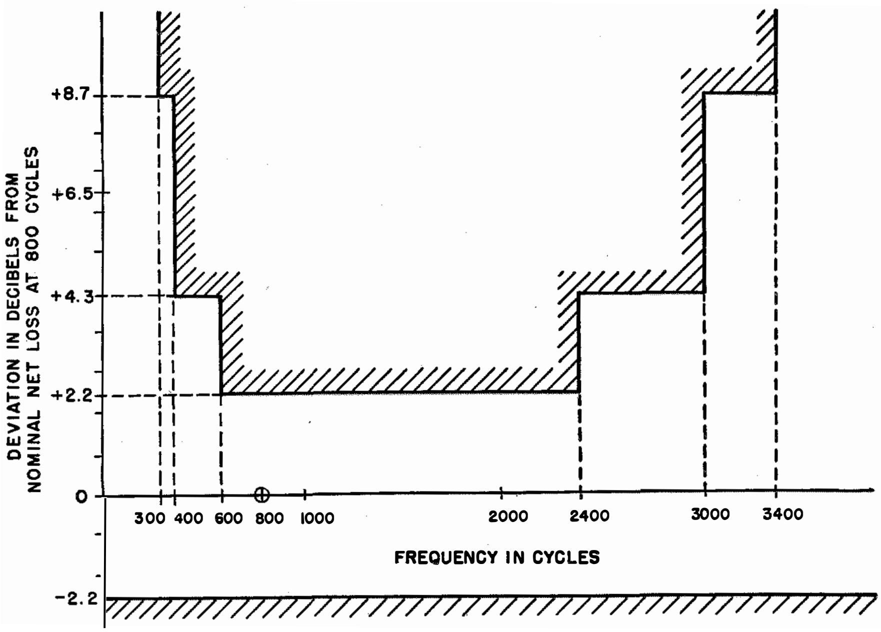

Fig. 5—Limits of loss-frequency characteristic of an international circuit effectively transmitting a band from 300 to 3400 cycles.

frequency is not more than 8·7 decibels above the nominal net loss of the circuit at 800 cycles. Fig. 5 shows the limits to which the frequency–net-loss characteristic of any international circuit in terminal service should conform. It is expected that modern equipments will provide circuits that will meet these limits with ease where there are no intermediate demodulation and modulation processes. To take into account, however, the future probability that long-distance international circuits will be built up from coaxial channels having such intermediate modulation and demodulation processes, the limits have been made less severe than would otherwise have been imposed. Administrations are asked to interpret the limits accordingly, and it is thought that they should be revised later when more experience is available on the operation of coaxial circuits.

D. *Echo Effects:* It is recommended that echo suppressors should not be inserted in the principal circuits of the international network and if echo suppressors are judged to be necessary on secondary circuits, that they be associated with the 4-wire terminating sets.

E. *Stability:* The stability of a built-up connection consisting of national and international toll circuits should not be less than 1·7 decibels calculated in accordance with Comité Consultatif International Téléphonique procedure.

F. *Propagation Time:* In designing the Toll-Switching Plan for Europe, the propagation time allowance should not exceed 50 milliseconds for each national system and 100 milliseconds for the international connection comprising two international circuits in tandem.

G. *Phase Distortion:* The limits of phase distortion would require to be given in considerable detail to be of any value. Full information may be found on page 34 of the "General European Telephone Switching Plan (1947–1952) Montreux 1946."

H. *Crosstalk and Noise:* The near-end or far-end equal-level crosstalk between two telephone circuits in the same cable should not be less than 58·2 decibels for 90 per cent of the possible combinations or less than 52·1 decibels for 100 per cent of the possible combinations.

These figures refer to 12- or 24-channel carrier-on-cable circuits and to coaxial circuits in terminal service.

The noise limits are still under study.

I. *Impedance:* In the interests of stability, it is recommended that all carrier terminal equipments in the future should have nominal input and output impedances of 600 ohms. This applies equally to national and international circuits.

J. *Additional Recommendations:* In addition to the foregoing recommendations, a considerable amount of work has been completed on detailed specifications for carrier and coaxial cables, carrier and coaxial equipment, and for broadcast and television circuits.

6. Acknowledgment

The author is indebted to members of the technical staff of the International Telephone and Telegraph System laboratories for valuable suggestions in connection with this paper.

System of Air Navigation and Traffic Control Recommended by the Radio Technical Commission for Aeronautics

By P. C. SANDRETTO

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TRANSPORTATION by aircraft of passengers and freight has increased to an extent where present methods and equipment for controlling air traffic are inadequate, particularly near major airports serving the leading cities in the United States. The Radio Technical Commission for Aeronautics has set up two programs, one of which is of an interim nature and seeks maximum utilization of existing facilities, while the other is of long-range significance and will require new methods and means to insure safe and expeditious flights from one airport runway to another, including maneuvering of aircraft on the ground. This program is expected to require about fifteen years for its completion and is discussed in this paper.

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Termination of hostilities marked the availability of unrestricted airline service to the general public in the United States, and since that time there has grown a deep realization of the inadequacy of the systems used for air navigation and traffic control. The congestion in areas surrounding airports that serve larger cities (for example, La Guardia Field, New York City) has sometimes caused aircraft to circle in holding stacks for several hours before they were permitted to land. It has been stated that under weather conditions necessitating flight by instruments, it has been necessary to cancel a large percentage of the schedules normally operating into La Guardia Field. A study made by the Air Transport Association showed that in 1946 because of canceled schedules due to low ceilings, reduced loads due to unreliability, and lost time due to airport congestion, the United States airlines lost a potential revenue of $39,500,000.

Not only were the aviation experts and those directly concerned with airline operation aware of the unsatisfactory systems of air navigation and traffic control, but the public was made to realize these conditions through articles in the general press.[1–3] It is most unusual for the public to take so much interest in such a technical subject. The Congress of the United States recognized the importance of an adequate solution to the problem of air navigation and traffic control, and made inquiries in hearings,[4,5] which took place in the winter of 1946. The opinions expressed at these hearings were, however, so diverse that they did not provide Congress with a suitable basis on which enabling legislation could be passed. As a consequence, the Radio Technical Commission for Aeronautics set up a special group to develop the broad principles of a system of air navigation and traffic control and to make recommendations for suitable instrumentation.

1. *Radio Technical Commission for Aeronautics*

The Radio Technical Commission for Aeronautics is the only organization in the United States on which are represented the Air Force, Navy, Civil Aeronautics Administration, and all other cognizant governmental bodies, together with civil agencies such as Aeronautical Radio, Incorporated, Air Transport Association, Airline Pilots Association, Aircraft Owners and Pilots Association, and the aeronautical radio

[1] "What's Wrong With the Airlines," *Fortune;* August, 1946.

[2] Wolfgang Langewiesche, "Is the Air Full," *Harpers Magazine;* July, 1946.

[3] R. E. Nealy, "For Landing's Sake," *Colliers Weekly;* January 3, 1948.

[4] John H. Frederick, "Safety in Air Navigation," House of Representatives Committee on Interstate and Foreign Commerce; July 24, 1947.

[5] "National Aviation Policy," Report of Congressional Aviation Policy Board, Congress of the United States; March 1, 1948.

manufacturing industry. The Commission operates as an advisory body to the Air Coordinating Committee, which is the official implementing agency of the government. Meetings on the air-navigation and traffic-control problems were held in July of 1947 and the final report[6] was not available until the following February. The report contains two essentially separate recommendations: The first centers around equipment that is obtainable at present or that will become available in the near future; The second deals with a completely new and integrated system on which development is to begin immediately and which is expected to come into general use within a period of 15 years. Only the second recommendation will be discussed in this paper.

2. *Definitions*

Before discussing the recommendations, it may be well first to define *air navigation* and *traffic control.*

2.1 Air Navigation

Navigation is usually defined as the process of finding the position of a vehicle and directing it to a desired destination. Navigation may be based on one or more of four fundamental operations: dead reckoning, position fixing, pilotage, and homing. The aviation navigational system must solve the problem in three dimensions. The system must not only be based on one or more of the four principles, but must operate under all weather conditions and in such a manner as to safeguard the vehicle against unsafe deviations from course.

2.2 Traffic Control

Air-traffic control, which is usually defined as the process of providing for the safe and expeditious movement of aircraft, is considered to consist of four basic operations: introduction of aircraft into an air-traffic system, reservation of air space as a function of time, analysis of traffic patterns at all times, and issuance of traffic instructions.

The air-traffic-control system must not only provide safety by reserving inviolably a given air space to a given aircraft for a given period of time, but must make the space reservations in such a manner that the aircraft will find it unnecessary to "hold." The air-traffic-control system must allow for maximum utility of air space and runways. If the exact position of the aircraft as a function of time can only be determined to within one minute and the aircraft is traveling at a rate of three miles per minute, it is necessary to reserve a minimum space of six miles; therefore, for an air-traffic-control system to utilize air space with maximum efficiency, it must be imbued with the ability to determine location of all aircraft as a function of time with a high degree of accuracy and then be capable of issuing traffic instructions so that a minimum amount of time is consumed in their transmission and execution.

Now that *air navigation* and *traffic control* have been defined and some of the criteria for a good system have been enumerated, it is interesting to examine the methods utilized in the United States at the present time.

3. *Present Equipment for Air Navigation and Traffic Control*

The present system used in the United States cannot be said to have been planned. As each portion of the air-navigation and traffic-control problem became pressing, some action was taken to satisfy the immediate demand without any effort being made to coordinate with a possible future need. Air navigation is still based largely on the four-course low-frequency radio range, which was originally designed simply as a means for furnishing guidance. Later, an attempt to rectify some of the inadequacies of the range was made by adding 75-megacycle-per-second markers and airborne automatic direction finders. The very-high-frequency omnidirectional range now being installed was conceived largely as a static-free substitute for the low-frequency radio range. The concept of the omnidirectional range as a device that could do more than provide simple radial guidance came later and did not emerge until after the war. Instrument landing equipment could be discussed in a similar vein. In each case, the equipment in use was a spot solution for a spot problem, and no thought of integration was present. As a result, it is neces-

[6] Paper 27–48/DO-12, Radio Technical Commission for Aeronautics, Special Committee—31, "Air Traffic Control"; May 12, 1948.

sary for aircraft to carry a large number of equipments and still obtain inadequate service. It has been estimated that if the present trend continues, aircraft will soon be carrying 14 separate items of radio equipment to provide adequately for air navigation and traffic control.

While certain equipment is used for traffic control, it is not strictly true that this is in fact "traffic-control equipment." The traffic-control system operates on the basis of spot position reports derived from the obsolete navigational equipment previously described and transmitted over a communication system originally installed for the purpose of notifying the Post Office Department of the pasture in which the aircraft had made an emergency landing. To this conglomeration of equipment, there were added some hand-written pieces of paper that are displayed in the traffic-control room and analyzed by a group of men as the basis for determining proper traffic instructions. That the system has worked with a good safety record is indeed a tribute to the men who are responsible for its operation.

4. *Principles Set Up by Radio Technical Commission for Aeronautics*

The Radio Technical Commission for Aeronautics initially set up 17 broad principles that a traffic-control system must meet. These are as follows.

A. *Safety.* Safety in air operations shall be maintained and a positive means of providing for adequate separation of aircraft shall be incorporated within the system.

B. *Expeditiousness.* Means for expediting the movement of aircraft to the limit imposed by safety considerations shall be incorporated.

C. *Reliability.* So far as possible, the system shall be independent of weather conditions, equipment failure, or human factors.

D. *All Aircraft Use.* All types of aircraft shall be capable of using the system consistent with the operational characteristics of such aircraft.

E. *All Weather.* Weather conditions shall not bring about ineffective functioning of the system.

F. *All Air Space.* The system shall be capable of being readily expandable so that it may provide service in all usable air space.

G. *Minimum Control.* The amount of restrictive control applied to the aircraft shall be minimized.

H. *Integration.* Navigational and communication functions shall be related so as to constitute an integrated system.

I. *Human Factors.* For satisfactory operation, the system shall require the minimum in such human factors as consistency of training and alertness of air and ground personnel.

J. *Evolution.* The design shall be such as to permit progress in an orderly manner from the present equipment to that required in the eventual system.

K. *Flexibility.* It shall be practicable to handle a wide variety of operational situations.

L. *Security.* Due consideration shall be given to security in the interest of national defense.

M. *Limitations of Traffic Flow.* The system shall not constitute the limiting factor in determining the amount of traffic that can operate effectively in a given area.

N. *Language Difficulties.* Knowledge of a specific language shall not be required for the operation of the system.

O. *Identification.* It shall be possible to identify all aircraft immediately.

P. *Division of Responsibility.* The system shall be so designed that air-traffic control (planning, collection of information, and dissemination of traffic clearance) shall be the responsibility of a ground control agency with the pilot having the responsibility for complying with the control-agency clearance, subject to a primary responsibility of prevention of collision by any means at his disposal.

Q. *Division of Equipment.* The primary burden of site, volume, and physical complexity of equipment shall rest with the ground components.

A special committee, responsible for equipment implementation, used the foregoing principles as a basis to evaluate the present system. It was found to have a figure-of-merit of only 40 percent. Following this evaluation, all of the various proposed systems were added and it was found impossible by any means to select a system that has a figure-of-merit of more than 70 percent. It was the discovery that the mere adding of newer equipment would not produce a high-efficiency system (when weighed against the 17 principles) that prompted the Radio Technical Commission for Aeronautics to recommend a new system that could be devised only by an extended development program.

While a 15-year period appears to be a long time in which to bring into full utilization a new system of air navigation and traffic control, this time period is consistent with experience in other communication fields. For example, throughout the world there is now being installed a system of carrier transmission on coaxial cable, the planning for which began fully 20 years ago. It seems clear that, if aviation is to have an air-navigation

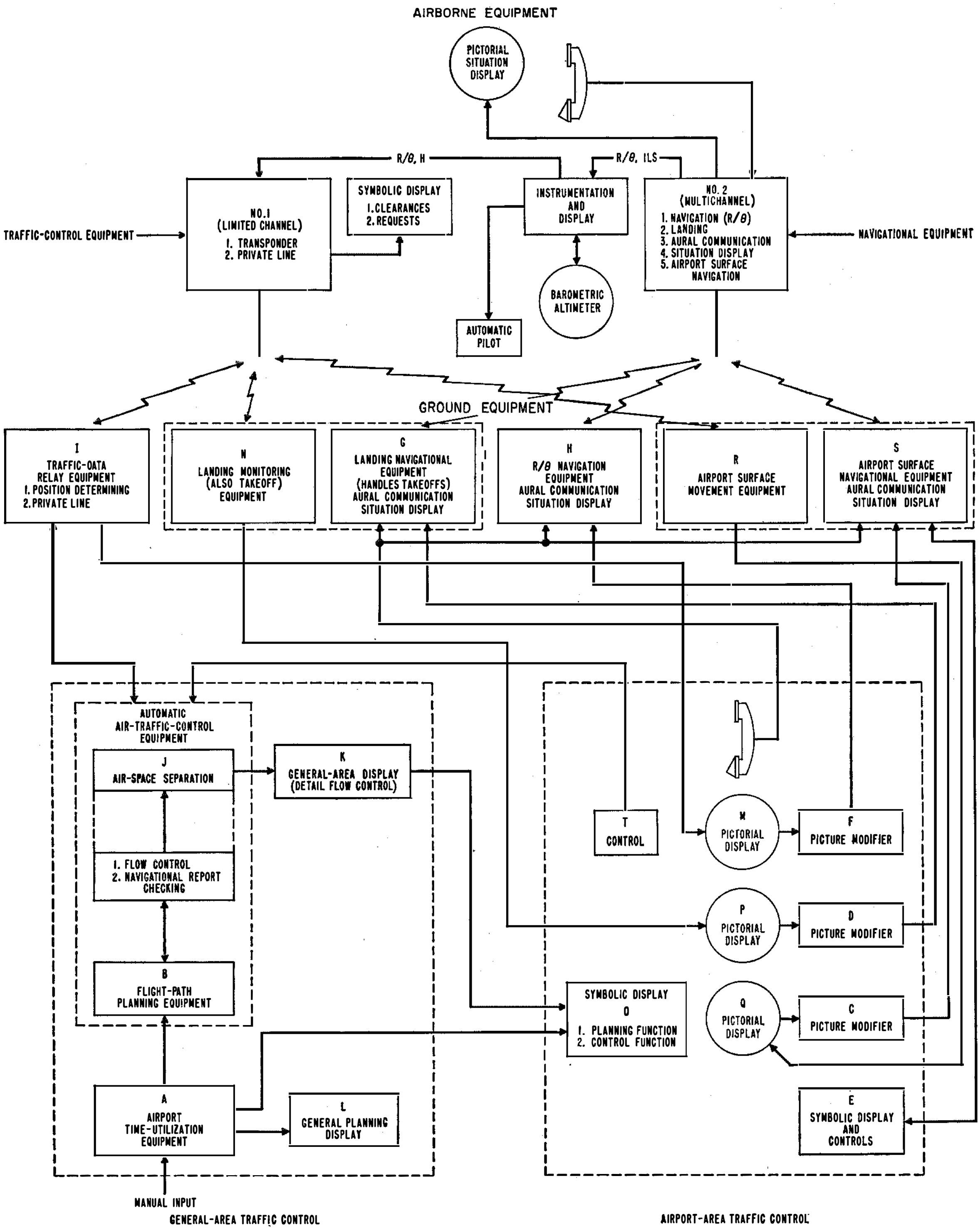

Fig. 1—Proposed system of air navigation and traffic control.

and traffic-control system that meets all the principles that have been established, it can only be brought about by long-range planning.

5. *Target System*

The so-called "target system" recommended by the Radio Technical Commission for Aeronautics will now be described. It is shown diagrammatically in Fig. 1. Essentially it consists of the following elements:

A. Two pieces of airborne equipment.

B. Four sets of ground equipment.

C. Equipment for use by two types of ground traffic-control agencies.

To analyze the system, it is broken down into some simplified block diagrams in Figs. 2, 3, and 4.

5.1 NAVIGATIONAL EQUIPMENT

Fig. 2 shows the principal elements proposed for navigational use.

5.1.1 Ground Equipment

Three sets of ground equipment are provided as follows.

A. Equipments *H* and *I* provide navigation for aircraft during the time they are operating in the *short-range enroute zone* (up to 200 miles between navigational facilities). Equipment *H* is responsible for the emission that brings data to the pilot. Its companion equipment *I* brings similar data about the aircraft to the ground controller without pilot assistance. The information furnished by both *H* and *I* is in the form of relative bearing and distance between the ground facility and the aircraft. A system providing bearing and distance is known as $\rho\theta$ (rho-theta) navigation and is the traditional form (similar to latitude and longitude). It is expected that the navigational information from *H* and *I* will be accurate to within 0.1 mile and 0.5

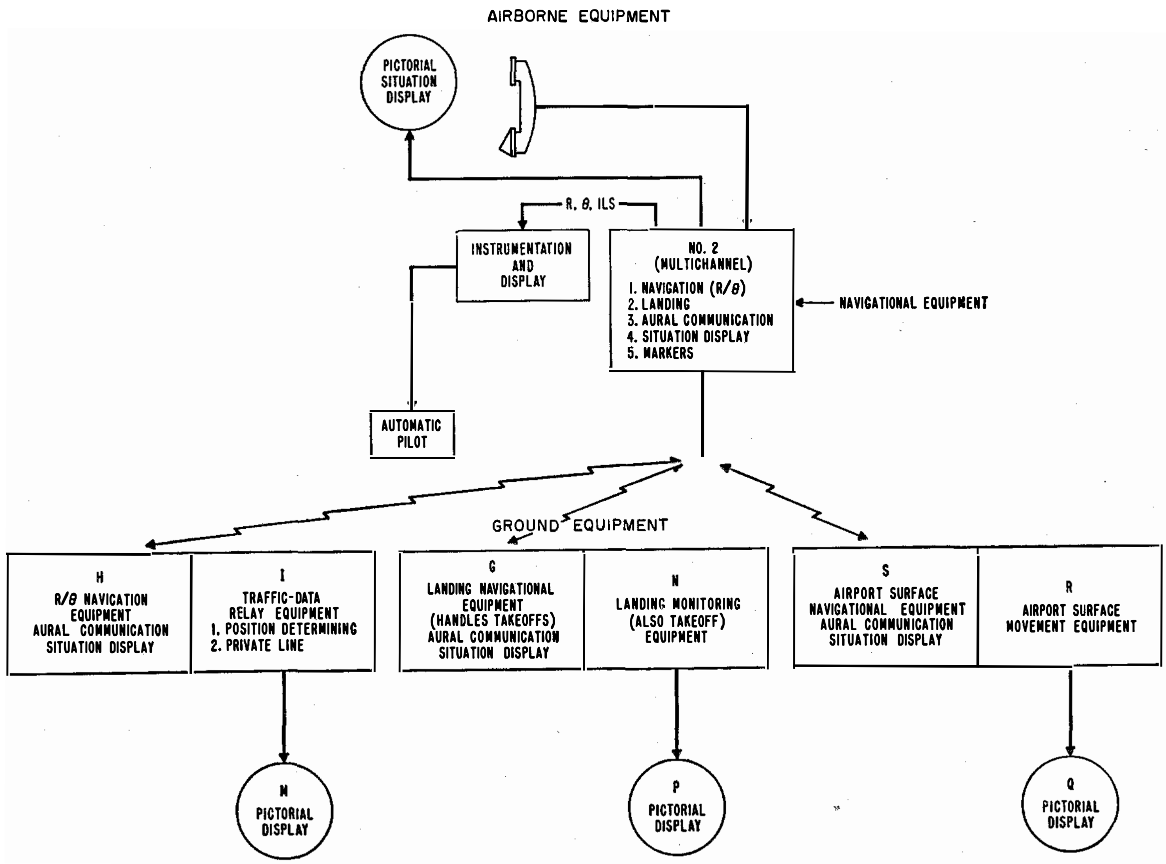

Fig. 2—Principal equipment for navigation.

degree. Equipment of this character will be installed along the airways at intervals of approximately 100 miles.

B. Equipments N and G provide navigational guidance when the aircraft is in the airport zone (within 30 miles). Equipment G provides the pilot with knowledge of the displacement of his aircraft relative to a landing path, and equipment N furnishes essentially the same information to a ground controller without pilot assistance. These equipments indicate quantitatively any displacement of the aircraft in the horizontal plane as small as 0.25 degree to the left or right of the center approach line of the path as measured from its point of origin. They also indicate any displacement of the aircraft in the vertical plane of the order of 0.05 degree above or below the path. At least one pair of equipments N and G will be used to an airport or perhaps one pair to a runway depending on the character of the finally developed equipment.

C. Equipments S and R provide guidance while the aircraft is on the surface of the airport. Equipment S is to furnish the pilot with information allowing him to navigate along the runways and to the loading dock, or from the loading dock to his point of takeoff. Equipment R is for furnishing the airport area controller with knowledge of the position of the aircraft on the surface of the runway. The exact character of equipments S and R is yet to be determined. It is expected that one pair of this equipment will be installed per airport.

There is no provision for navigational equipment when the aircraft is in the long-range enroute zone. Committee SC-31 of the Radio Technical Commission for Aeronautics did not consider the long-range problem. In addition to the navigational equipment shown in Fig. 2, long-range equipment must be provided for international aircraft, but the aforementioned group concentrated on the high-traffic-density problems, which are not now present in the long-range enroute zone.

5.1.2 *Airborne Equipment*

Indications from all ground navigational equipment are received in the aircraft via a single equipment shown as No. 2 in Fig. 1. This is multichannel equipment and while the exact number of channels was not determined, it is probable that there will be about 60. This equipment is to operate in the band from 960 to 1215 megacycles, although there may be some auxiliary transmissions at 3000, 5000, or 10,000 megacycles. While it is hoped that by extended development (taking place over a period of years),

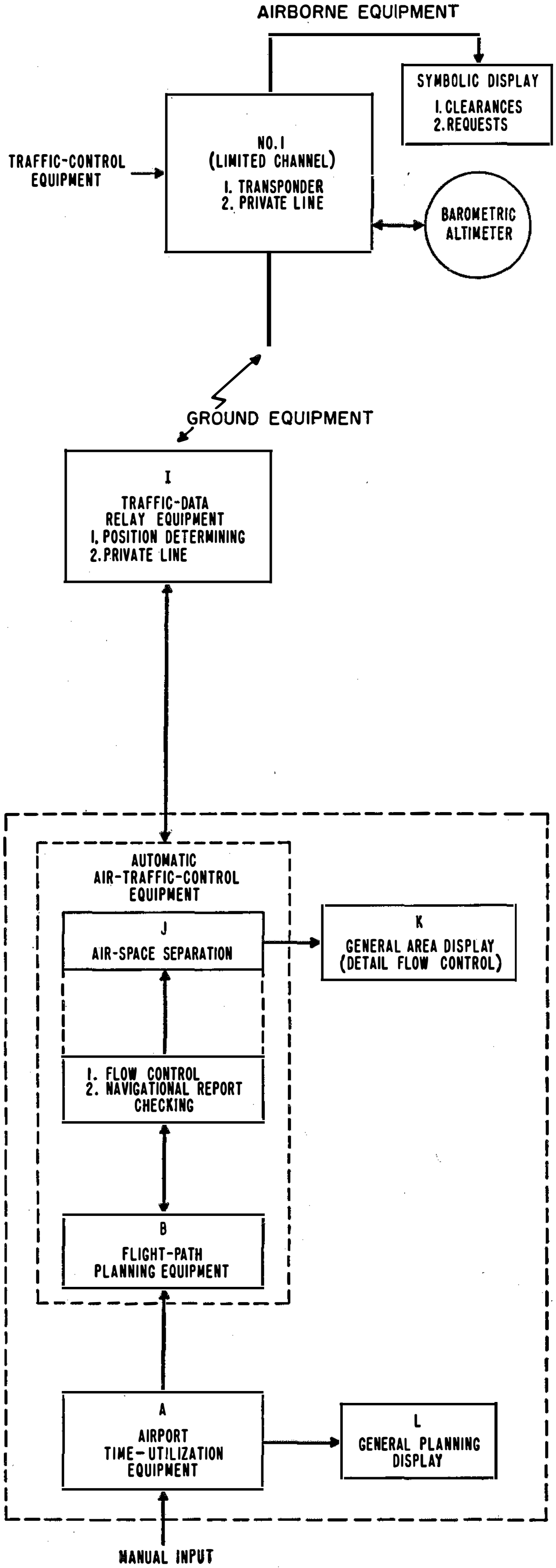

Fig. 3—Principal equipment for general-area traffic control.

the weight of the No. 2 equipment (minus some indicator units) can be reduced to only 20 pounds, it is expected that this single equipment will furnish the following facilities.

A. Measurement of the distance of the aircraft to the selected ground beacon.

B. Bearing of the aircraft to the selected ground beacon.

C. Displacement of the aircraft with respect to a localizer course or courses.

D. Displacement of the aircraft with respect to a glide slope.

E. Display of the general traffic situation.

F. Indication of the position of the aircraft over certain definite points on the ground where radio marker beacons are located.

G. Two-way voice communication.

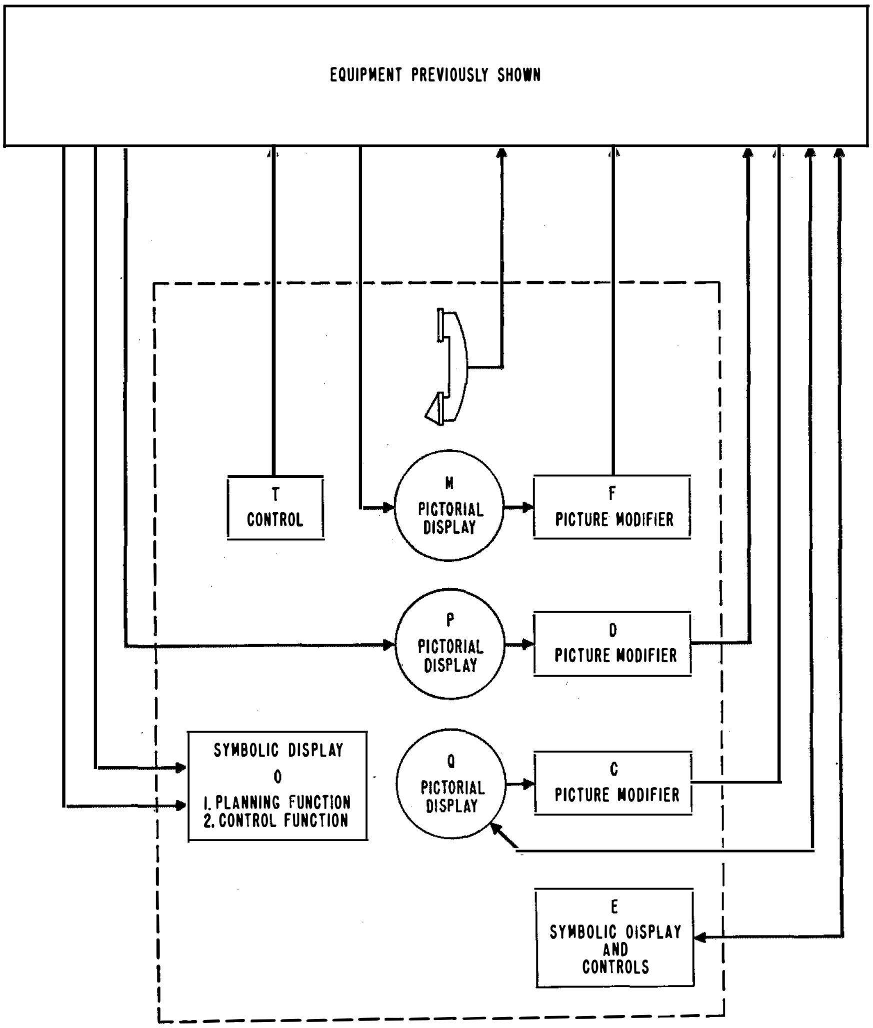

Fig. 4—Principal equipment for airport-area traffic-control office.

The automatic operation of the aircraft by use of the No. 2 equipment is provided through a unit marked "instrumentation" in Fig. 1. This instrumentation unit will incorporate means for indicating the bearing and distance of the aircraft with respect to the ground navigational station. It also provides means whereby the aircraft can be flown on any one of a number of parallel tracks during the time it is in the enroute zone.

The pilot can determine the position of the aircraft by indicators that form part of the instrumentation unit but he also is able to obtain information through the pictorial display. This pictorial display is expected to be of particular assistance during the time that the aircraft is approaching the runway for landing and may be the primary source of navigational information when the aircraft is on the surface of the runway.

To summarize, the navigational equipment was planned so that it will furnish the following types of service within three zones where aircraft operates.

A. Means whereby the aircraft can be guided automatically along arbitrary courses of known coordinates.

B. Means whereby the pilot can know his position and thus be able to take over the guidance manually.

C. Means whereby ground personnel can, independently of the pilot, determine the position of the aircraft.

By performing the foregoing three functions through the use of weather-free transmission at very and ultra-high-frequencies, it is expected that the requirements for safety in navigation can be fully met.

6. *Traffic-Control Equipment*

6.1 General-Area Traffic-Control Equipment

To understand the provisions suggested for traffic control, it must first be understood that traffic is controlled by two different types of ground areas. There are 23 such areas in the United States. In each area there is a *general-area traffic-control office*, which is the agency responsible for the control of all traffic along the airways of that area. The general-area office also has the authority to control traffic when it is off the airways but seldom finds it necessary to act under these circumstances. The agency responsible for the control of traffic near and within the area immediately surrounding the airport is the *airport-area control office.*

The principles of traffic-control area offices and their zones of jurisdiction are illustrated in Fig. 5, which includes a portion of the United States with 10 general areas. For each general area, there is one general-area traffic-control office. This office is usually located at the most important city within the area. In addition, there are airport-area traffic-control offices located at each important airport within the general area.

The principal equipment provided for traffic control in the general area is shown in Fig. 3. In an idealized air-traffic-control system, the only limit to the amount of traffic that can be handled is the number of runways available and the length of time required for each aircraft operation to use a runway. The construction of runways is very expensive; therefore, it is highly important that a means be provided whereby runway time may be utilized with maximum efficiency. In recognition of the aforementioned principle, the general-area traffic-control office employs a unit known as the *airport time-utilization equipment.* This equipment is designed to assure that the total number of aircraft dispatched from all sources and scheduled to arrive at a certain airport during a given period of time will not exceed the number that can be safely landed within that period at the airport. Equipment *A* of Fig. 3 makes a time reservation for each aircraft to utilize the runways of an airport either for take-off or landing. For example, if an aircraft desires to arrive at New York at 10:15 but that time has already been allotted to another aircraft, equipment *A* finds the first available time and may reply that 10:17 is now reserved for the aircraft. This equipment assures that the available runway time will be utilized to maximum efficiency and there will be no delay in landings or take-offs so long as all aircraft adhere to the time schedule.

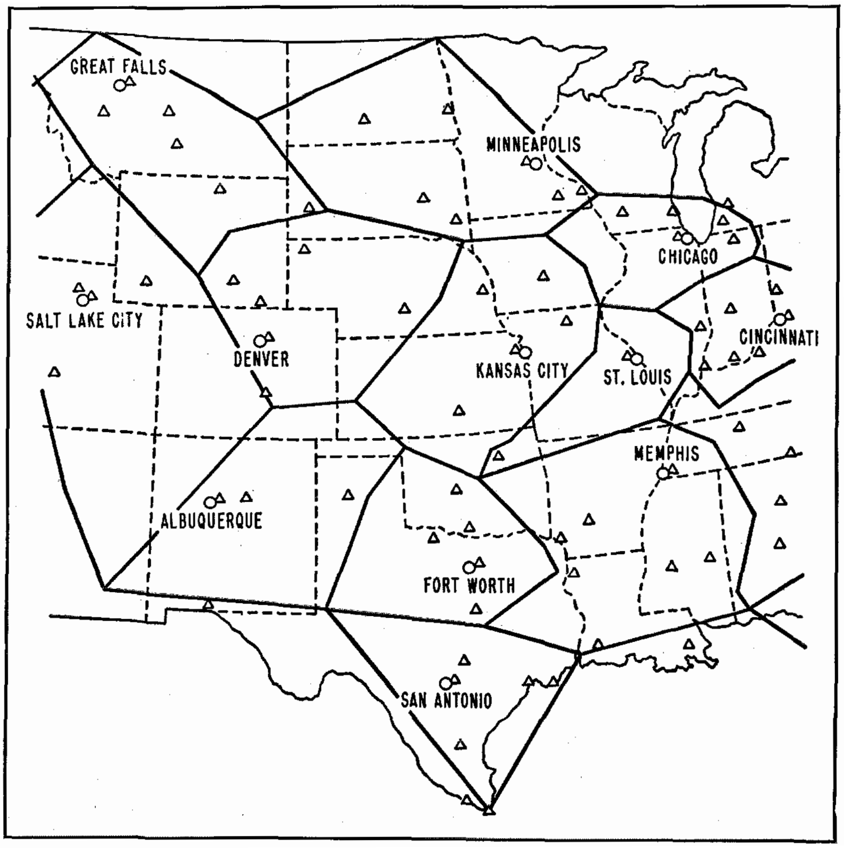

Fig. 5—Air-traffic-control areas. General-area offices are circles and airport-area offices are triangles.

The major function of equipment B, designated the *flight-path-planning equipment*, is that of assuring an expeditious flow of air traffic. If means were provided whereby a signal were transmitted when one aircraft approached another causing the aircraft to hold, the flight would be performed in safety but much time might be spent in circling instead of progressing towards the destination. Into equipment B is inserted the proposed flight plan of an aircraft; that is, the position in space it expects to occupy as a function of future time. Having this information, equipment B looks into the future to insure that two aircraft will not attempt to utilize the same space at the same time. It will approve a flight plan only if its calculations reveal that an aircraft can follow that plan without having to hold at any time. After the aircraft is in the air, equipment B is able to determine its performance with respect to its planned schedule and indicate whether the schedule is being maintained. As a separate task, it can set up a new schedule for the aircraft to follow if it is too far ahead or behind its original plan.

The instantaneous position of the aircraft is determined by I, the *traffic-data relay equipment*, which is an interrogator-responder.[7] The traffic-data relay interrogates the No. 1 airborne equipment. In response to the challenge, equipment No. 1 replies and immediately the range and azimuth of the aircraft are known. Equipment No. 1 has a complex response and furnishes the following information.

A. The altitude of the aircraft (as derived from a barometric element).

B. The identity of the aircraft in terms of its frame number.

C. The acknowledgment of safety signals indicating that the aircraft will proceed, change, hold, etc.

D. Request for flight-plan change in route, lane, altitude, time, speed, or destination.

Knowing the position of the aircraft, equipment J, the *safety separation equipment*, immediately indicates whether the aircraft may proceed or if it must hold because another aircraft is occupying the next block of air space of certain assigned dimensions. This function is known as "air-space separation," and is a safety provision. Like block signals on a railroad, the aircraft is informed that it is about to approach too close to another aircraft. Equipment J in conjunction with equipment I also sends the aircraft a clearance in response to a request. It compares the position of the aircraft as reported by the navigational equipment with that being determined independently by equipment I and reports any significant deviations.

The total output of equipment I furnishes the aircraft with the following information.

A. Safety signals indicating that the aircraft may proceed, must change its course, should be on the alert, should hold, or that the equipment is not operating.

B. Clearance signals permitting the aircraft to change its route, lane, altitude, estimated time of arrival or departure, and destination.

C. Error signals instructing the aircraft to go up or go down, go right or left, or change speed to maintain its time schedule.

It is envisaged that units of type I equipment will be located at intervals of about 100 miles and will be linked by microwave radio to the general-area traffic-control office in each general-type traffic area. In these offices there will be located the J and B equipments. Displays K and L assist in the planning of traffic flow and allow manual control in emergencies.

The No. 1 airborne equipment is to operate in the frequency range of 1365 to 1660 megacycles but there will be auxiliary transmissions in the 3000-, 5000-, or 10,000-megacycle bands. It is expected that this equipment can, by extended development, be reduced to a weight of 20 pounds.

To summarize, the general-area traffic-control system has provisions for the following services.

A. Means whereby a pilot can proceed only with the certain knowledge that his aircraft cannot occupy an area of a definite dimension that is already occupied by another aircraft. That is, he can proceed only if it is safe for him to do so.

B. Means whereby the necessity for holding has been investigated before an aircraft departs and the pilot can proceed with the knowledge that, if his aircraft adheres to the planned schedule, there will be no delays in the journey.

C. Means whereby runway time, the limiting factor in air operations, will be fully utilized.

[7] Wesley J. Leas, "Surveillance Radar Difficiencies and How They Can Be Overcome," *Proceedings of the I.R.E.*, v. 36, pp. 1015–1017; August, 1948.

By performing the foregoing three functions, it can be seen that the requirements for safety, expeditiousness, and maximum efficiency should be capable of being fully realized.

6.2 Airport-Area Traffic-Control Equipment

Before discussing the equipment provided for the airport-area traffic-control office, it may be well to consider the difference between the conditions existing in the enroute area and those in the general area. Broadly speaking, the purpose of both types of control is similar; to provide for the safe and expeditious movement of aircraft. However, the space available in the airport area is much less.

In the general area, control is applied to aircraft at altitudes below 20,000 feet, but in the airport area this is restricted to a maximum of 7,000 feet. The diameter of the airport varies but is never more than 30 miles, and 7 miles is a common figure. At some time *all aircraft* in the airport area must travel at one altitude and along one or two narrow lanes the—runways.

It can thus be seen that while the problems of both the airport and general areas are similar, the controls to be exercised by the former must be much more precise. In the past, this problem has been circumvented rather than solved by having aircraft circle in definite lanes and gain time in this manner. Since time is distance where moving vehicles are concerned, the practice of holding has made the area larger. This is not an academic premise as may be illustrated by con-considering a passenger who has spent an hour coming from one airport and then must circle for another hour in a holding stack in the vicinity of the destination airport. To him, it was as far from the holding stack to the runway as it was from one airport to the other.

All facilities for planning a safe and expeditious flight have been provided and described in connection with the general area. The airport area utilizes all these same facilities, but makes provisions whereby the airport-area control-office operator will have means for knowing accurately where all aircraft are located and for rapidly transmitting information to them. These facilities are shown in Fig. 4.

The airport traffic-control office is informed in advance of the appearance of aircraft in the airport area by the symbolic display *O*.

The airport-area traffic-control operator determines the position of the aircraft by displays of any one of three cathode-ray plan-position indicators. These plan-position indicators are shown as *M*, *P*, and *Q* in Fig. 4. Equipment *M* derives from equipment *I* the position of aircraft while in the enroute area. Display *P*, through equipment *N*, shows the position of aircraft while they are on the landing path. Display *Q*, through equipment *R*, shows the position of aircraft on the surface of the airport.

The airport-area traffic-control operator has three means at his disposal for issuing traffic instructions to aircraft. The handset shown indicates the voice communication facilities provided through navigational equipment *N*, *M*, or *S*, and the airborne Nav-aid equipment. By means of control *T* associated with the automatic air-traffic equipment, instructions may be sent to the pilot in symbolic form. These symbols would transmit such predetermined messages as "proceed," "hold," "descend," etc. Picture modifiers *F*, *D*, and *C*, associated with displays *M*, *P*, and *Q*, respectively, provide a means whereby instructions may be sent pictorially to the pilot. As an example of how picture modifiers may be used to send instructions, it will be possible to draw a line on the picture to be transmitted showing the path an aircraft should follow when it is on the surface of the airport. It can thus be seen that almost every conceivable means for sending instructions rapidly are provided, together with excellent means for knowing the position of all aircraft.

Perhaps it is too much to hope that any proposed traffic-control system will eliminate the need for all holding in the airport area. However, from the foregoing description it is apparent that the following facilities have been provided:

A. Means whereby holding can be conducted in safety.

B. Means for reducing holding times to a minimum.

7. Conclusion

In the foregoing, there was given the outline of a completely integrated system of air navigation and traffic control designed to meet a number of predetermined operational requirements.

The apparatus to fulfill the outline of the system is to be developed and placed in use over a period of 15 years. It is believed that this is the first time a system of air navigation and traffic control has been predetermined and then a program established to produce the equipment in accordance with the outline specification. The predetermination of the system is, indeed, a milestone in communication, as well as aeronautical, history.

Since only an outline of the system was prepared, it is impossible to give at this time the exact technical character and the design parameters of each of the apparatus units that will go to make the final system. As development progresses, many authors will, no doubt, prepare papers in which the detailed description of circuits and elements will appear. Because such information is not available today, however, does not mean that the Radio Technical Commission for Aeronautics system must wait on future invention before it can come into being. The war years have produced the basic techniques and the years since the war have witnessed many applications of these techniques to practical apparatus. In various laboratories, the embryos of many of the elements shown in Fig. 1 already exist. Only time is required to bring apparatus units into a completely developed form for blending into a system that fulfills the outline proposed by the Radio Technical Commission for Aeronautics.

8. Addendum

Federal Telecommunication Laboratories and Federal Telephone and Radio Corporation have been active participants in the development and manufacture of electronic aids to aerial navigation and traffic control. Significant contributions made by these two organizations to instrument landing,[8] distance-measuring systems,[9] and ground radar[10] may be considered to be part of the interim program. Contributions to the long-range plan have, of course, been restricted primarily to theoretical investigations; many of the concepts resulting from these studies[11] have been incorporated in the plan of the Radio Technical Commission for Aeronautics.

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[8] Sidney Pickles, "Army Air Forces' Portable Instrument Landing System," *Electrical Communication*, v. 22, n. 4, pp. 262–294; 1945.

[9] H. Busignies, "High-Stability Distance-Measuring Equipment for Aerial Navigation," *Electrical Communication*, v. 25, pp. 237–243; September, 1947.

[10] J. S. Engel, "Landing Aircraft with Ground Radar," *Electrical Communication*, v. 24, pp. 72–81; March, 1947.

[11] H. Busignies, P. R. Adams, and R. I. Colin, "Aerial Navigation and Traffic Control with Navaglobe, Navar, Navaglide, and Navascreen," *Electrical Communication*, v. 23, pp. 113–143; June, 1946; see pp. 136–143.

Application of Gas-Filled Tubes for Storage and Sending

By F. H. BRAY, D. S. RIDLER,
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ELECTRONIC MEANS are described for storing and retransmitting digital information in a form suitable for automatic telephone systems. The use of miniature cold-cathode tubes permits the construction of compact equipment to replace existing apparatus employing relatively large numbers of electromechanical relays. Both space and maintenance costs are reduced.

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The storage and subsequent retransmission of digital information is an essential feature in automatic telephone systems and allied fields. Existing methods usually employ a group of relays for the storage of each digit and a further group of relays or a selector for controlling the retransmission of stored information as trains of impulses. Such schemes invariably tend to produce equipments involving large numbers of relays with consequent space considerations while maintenance costs are high, especially on the transmitting device, which has a very heavy duty cycle. It is with a view to removing such objections that the use of gas-filled tubes for storage and sending purposes has been considered, especially in view of recent advances in the construction of miniature tubes and improvements in their performance.

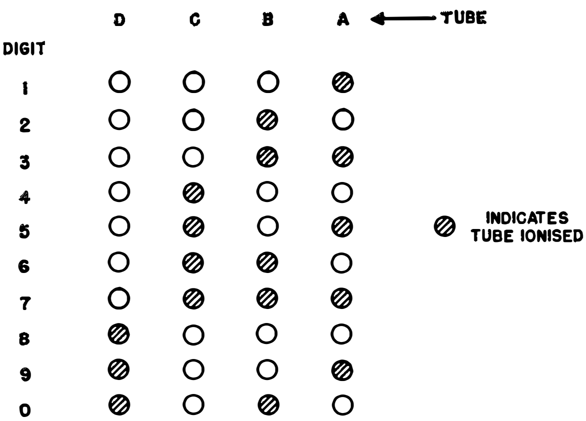

Fig. 1—Direct storage of numbers from operator's key set.

1. Types of Tubes Used

The following types of tubes are used for the functions indicated.

A. 2-electrode neon tubes are used as storage units.

B. Double-trigger cold-cathode miniature tubes are used as storage units and in counting trains.

C. Cold-cathode gas relay tubes are used as sensitive relays in storage, counting, and detecting applications where current is required.

D. Miniature cold-cathode gas relay tubes are used in storage elements where little current is required.

E. Miniature 50-volt stabilizer tubes are used as voltage limiters in counting trains.

2. Storage on Gas-Filled Tubes

It is well known that tubes filled with neon and like gases can have a characteristic such that the potential necessary for initiating a discharge is considerably in excess of the potential necessary for sustaining the discharge after ionisation has occurred. An applied operating potential between these so-called striking and sustaining potentials will therefore maintain a series combination of tube and resistance in one of two stable conditions, i.e., the tube de-ionised and therefore not conducting, or ionised and conducting. A tube discharge gap may be ionised by instantaneously raising the striking potential or quenched by momentarily lowering the potential below that required for sustaining the discharge. The condition of such a tube may be detected by sensing the potential drop across a series resistance. If V_o is the normal operating potential for the tube and V_s is the sustaining potential, then the potential across the series resistance is $V_o - V_s$ when the tube is conducting and zero when the tube is normal.

For the storage of digital information, a group of four tubes is necessary for each digit and will provide 16 possible combinations depending on which tubes are conducting or normal. It is found most convenient to store such information in a binary code for reasons that will later become

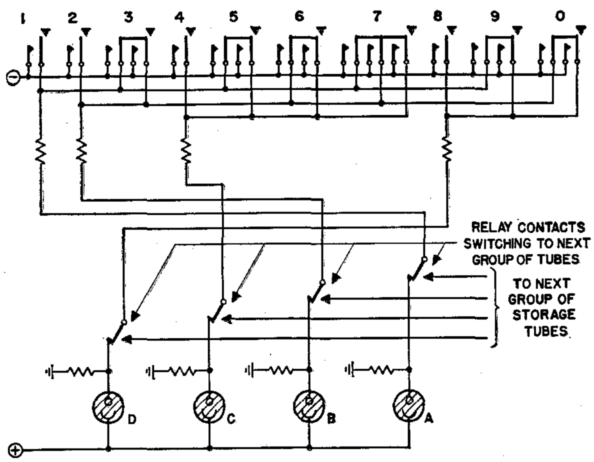

Fig. 2—Binary code basis of storing numbers.

apparent. Thus, the first tube ionised in a group of four tubes represents $2^0=1$, the second tube ionised represents $2^1=2$ and so on for successive powers of 2 until the fourth tube is reached. In practice, this process of storage can be very simple and Fig. 1 shows, for example, storage from an operator's key set. Respective tubes are ionised by a negative potential applied through suitable limiting resistances and the distribution between groups of tubes per digit is by means of relay contacts. Fig. 2 illustrates the storage in binary code. Fig. 3 shows a mounting designed for accommodating two sets of 2-electrode neon tubes. Each set consists of 4 tubes with their series resistors and together are capable of storing 2 digits. The complete unit mounts in the space required by one telephone-type relay.

3. Gas-Tube Counter

Use is made of the gas-tube binary counter for the translation of digits stored in binary code into decimal trains of impulses and for like functions. The counter incorporates double-trigger miniature cold-cathode tubes, each having an anode, cathode, and two similar but independent trigger electrodes. The tubes are of miniature construction and when used in a circuit with a main-gap current of the order of 0.5 milliampere give satisfactory life.

The tubes are used in pairs in "flip-flop" circuits coupled together to give binary counting action. Only one tube in a pair is normally ionised and ionisation of the other causes de-ionisation of the first. A train of impulses is applied to the first pair such that each pulse is passed forward to the next pair of tubes for each alternate applied pulse. The changeover of discharge, therefore, takes place at only half the rate on the second pair of tubes. In the same way, a pulse is passed forward to the third pair of tubes at each fourth applied pulse to the counter and to a fourth pair of tubes at each eighth applied pulse. Fig. 4 gives a typical circuit of such a counter.

It can be seen that a counter consisting of n pairs of tubes will have a capacity of 2^n, four pairs requiring $2^4=16$ impulses to complete each counting cycle.

There are several methods of coupling pairs of tubes together. Transformers or rectifiers may be used and direct coupling is possible given suitable gas-tube characteristics and design. However,

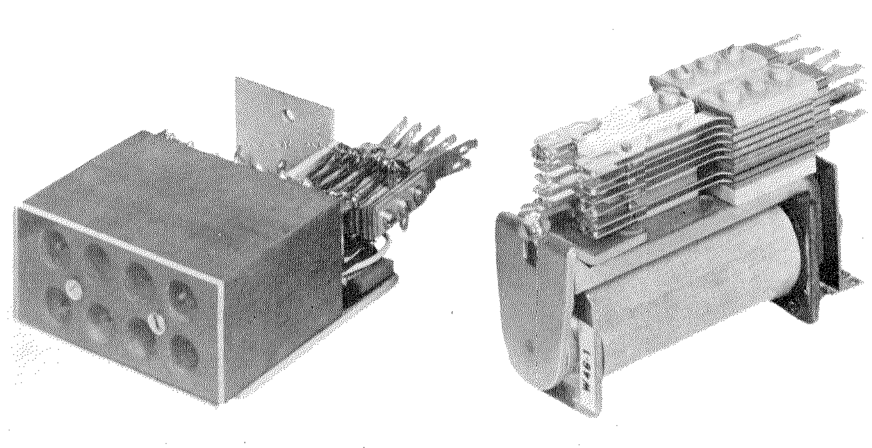

Fig. 3—Assembly of 8 neon tubes with series resistors to store 2 digits is at left. A telephone-type relay at right occupies the same amount of space.

one of the simplest and most economical means is shown in Fig. 4 in which a miniature 50-volt neon stabiliser tube provides the coupling. In detail, the operation is as follows.

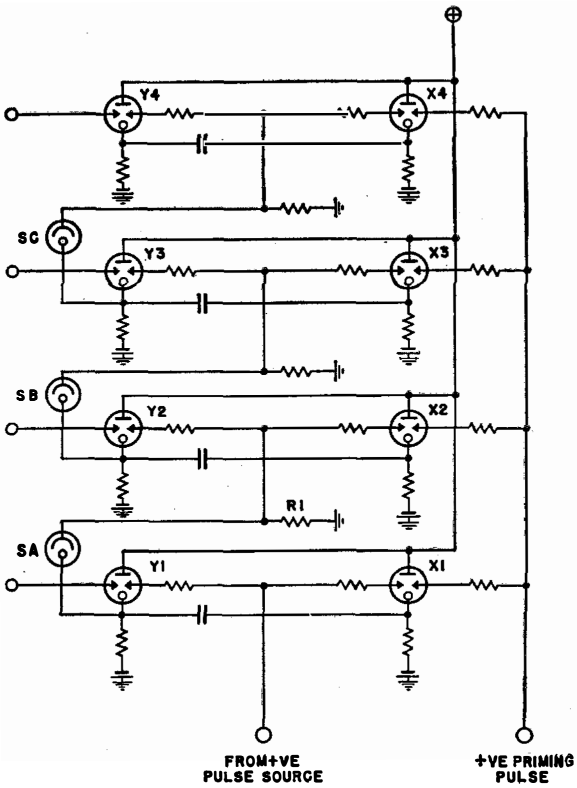

Fig. 4—Flip-flop circuit used as a binary counter.

A priming pulse is momentarily applied to the control electrodes of all the *X* tubes in the counter to establish the normal or start conditions. All *X* tubes sustain ionisation across their main gaps. The first pulse to the counter from the pulse source causes *Y1* tube to ionise and its cathode potential rises from −50 volts to approximately +50 volts since the main-gap sustaining voltage is of the order of 80 volts. This rise in potential on one side of the cathode coupling capacitor momentarily raises the cathode potential of *X1* and this tube de-ionises as there is insufficient potential to maintain a discharge. Fig. 5 gives a graphical indication of the potential changes.

After receipt of the first impulse, the counter is therefore left with *Y1*, *X2*, *X3*, and *X4* tubes ionised. The second impulse causes *X1* to re-ionise, which causes *Y1* to de-ionise and a pulse is passed on to the next pair of tubes since the neon stabiliser *SA* conducts, stabilises at 50 volts, and leaves a +100-volt *VE* pulse across *R1*. Tube *Y2*, therefore, ionises and in turn de-ionises *X2* as already described. The receipt of two impulses therefore leaves the counter in the condition of *X1*, *Y2*, *X3*, and *X4* ionised. The process continues in this manner with the de-ionisation of any *Y* tube in a pair causing a pulse to be passed on. The receipt of 16 impulses

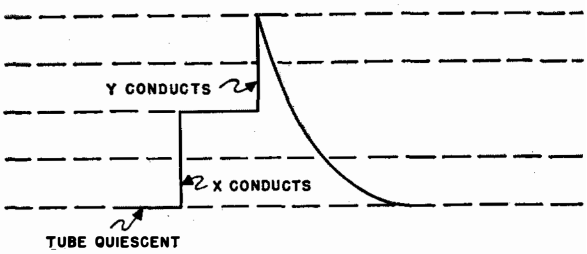

Fig. 5—The incoming pulse causes tube *Y* to conduct after the ionization potential is exceeded. The exponential decay is caused by the discharge characteristics of the coupling capacitor and cathode resistor.

brings the counter back to normal or the start condition and the cycle of operations is then repeated. An impulse resulting from the ionisation of *X4* can be used to indicate to an external circuit that the cycle of operations has been completed.

Counters of this type have been made as plug-in units in an extremely compact form. Maintenance is made easy by virtue of visual observation of the discharge in the tubes and hence of any stored number. Current consumption is only a few milliamperes. Fig. 6 is a view of a typical plug-in counter. Fig. 7 is a view of a sender

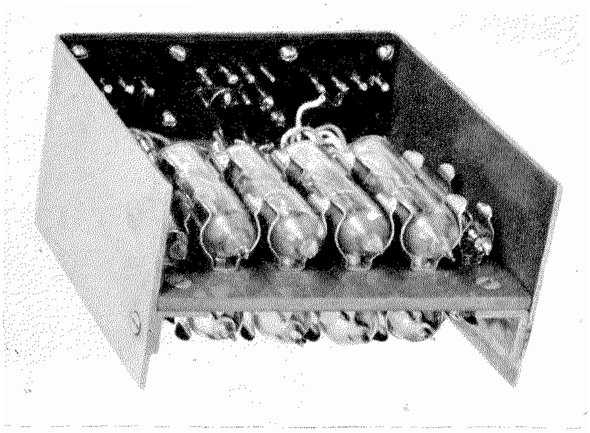

Fig. 6—Plug-in counter.

capable of storing and transmitting 8 digits, the storage units and counter being seen at the left of the equipment.

4. Association of Storage with Counting

The type of counter just described provides the essential link between trains of digital impulses and corresponding numbers stored on four tubes in binary code. The transformation can be made either way enabling digital information to be put into or taken out of storage as required. A simple means of storage on sets of four neon tubes has been described in Section 2 and the production of trains of digital impulses corresponding to keyed numbers provides a typical example of the use of storage and counting in combination. It has been shown that the counter completes a cyclic operation with each sixteenth impulse and that, assuming the first impulse arrives when the counter is normal, the ionisation pattern of *Y* tubes represents the number of impulses that have been applied to the counter. It follows therefore that, if the *Y* tubes are set in a pre-determined pattern before impulses are applied to the counter, the cycle can be completed before the sixteenth impulse, in fact the cycle is complete when there is a number of impulses equal to 16 minus the number represented by the pattern of pre-ionised *Y* tubes. If, say, the *Y* tubes are set to 1010 indicated by *Y4* and *Y2* ionised, the binary equivalent of 10, then only 16−10 or 6 impulses will be required for the counter to complete a cycle. Fig. 8 shows a means for achieving this.

Each neon storage element is connected to the control electrode of its corresponding *Y* tube in the counter by means of relay or switch contacts or later by electronic means, and the potential across the series resistance of an ionised storage tube will trigger the *Y* tube in the counter. No pulse is passed on in the counter when a *Y* tube ionises and so no interference between pairs of tubes in the counter can take place. After the counter has been pre-set in this way, a train of pulses at standard impulsing speed is applied to the first pair of tubes and at the same time

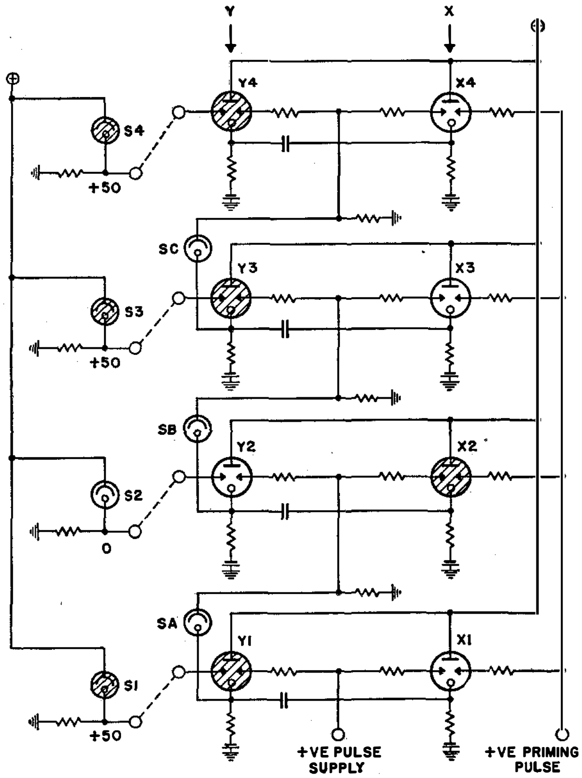

Fig. 8—Transfer of stored digit into counter.

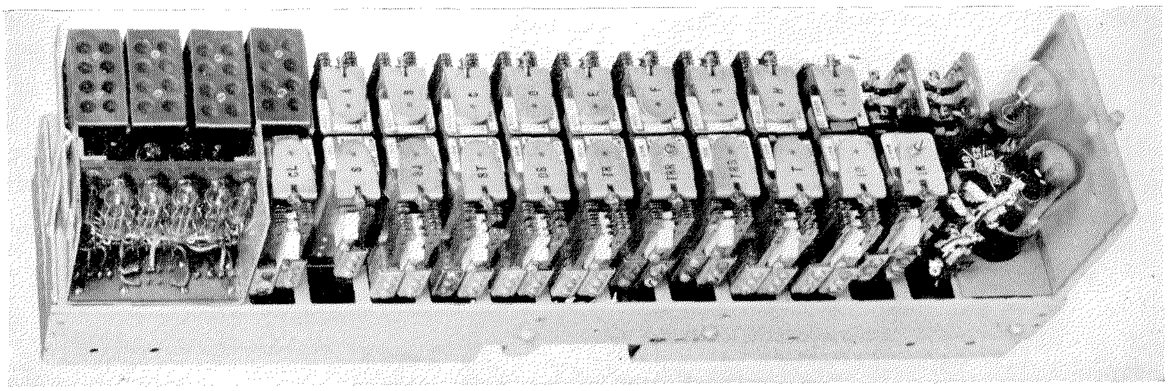

Fig. 7—Sender capable of storing and transmitting 8 digits.

impulses are sent out to the switching equipment. Transmission of impulses ceases when the counter has completed its cycle and restores to normal, a condition which is indicated by a pulse passed forward by the last pair of tubes when *X4* ionises. In this way, the number of impulses represents, as explained, the complement of the stored number with respect to 16 and so determines the original pattern that must be applied from the operator's key set to the storage tubes.

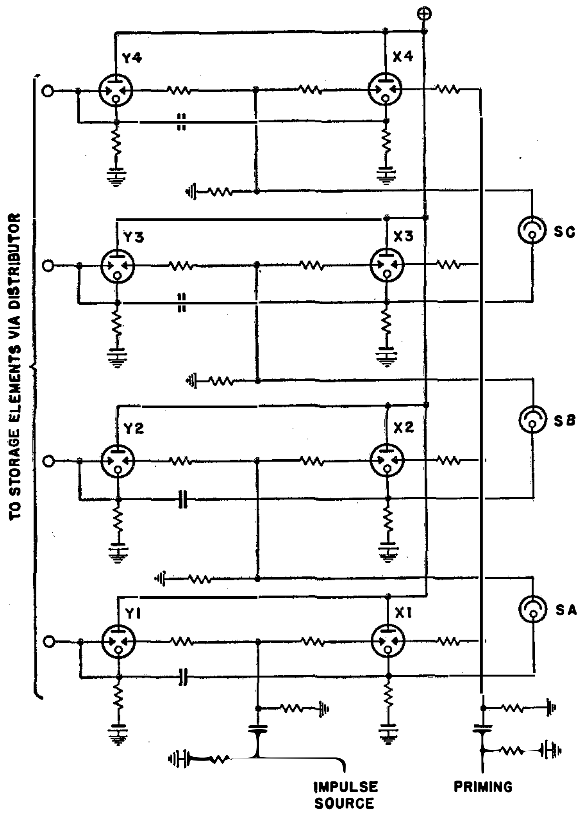

Fig. 9—Binary counter for reception of dialled impulses.

5. *Reception of Dialled Impulses and Storage*

In registers, for example, where trains of impulses are received and stored, it follows from Section 4 that such trains should be stored not as the binary equivalent but as the binary complement with respect to 16 so that re-transmission from the storage elements becomes a simple

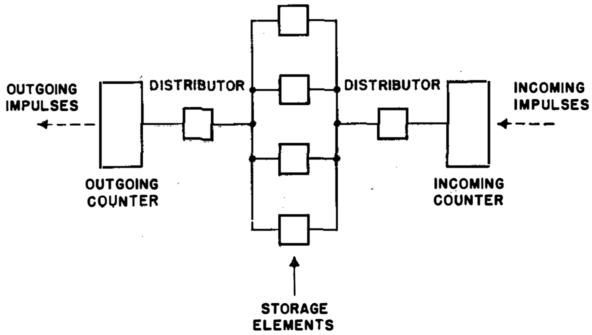

Fig. 10—Block diagram of general scheme applied to a register.

process as just described. In general, the dialled impulses are stored on an incoming counter as the binary complement with respect to 16, by a slight modification to the counter as already described and as shown in Fig. 9, and then transferred to storage elements, the storage tubes being ionised via a distributor from the positive potentials present on the *Y* tubes that are conducting in the counter. For re-transmitting the stored digits, the storage elements are then connected in turn to an outgoing counter that will transmit the original dialled digits since the storage is the binary complement of the digit with respect to 16, say *X*, and hence the counter will transmit 16 minus *X*, the original dialled number. In Fig. 8, the *X* tubes are all ionised prior to the reception of dialled impulses and the first impulse to arrive will trigger *Y1* which de-ionises *X1*. A pulse is passed to the next pair of tubes in a manner as described for Fig. 5 causing *Y2* to ionise followed by *Y3* and *Y4*. Thus, if the dialled digit were *1* then the counter is left with all *Y* tubes ionised, which in turn will ionise all 4 storage elements and represents 15. If the digit were 5, then *Y4*, *Y2*, and *Y1* are ionised corresponding to 11, which is passed to the storage tubes. Fig. 10 illustrates in block schematic form the general scheme as applied to a register.

Electrical Properties of Plastics*

By A. J. WARNER

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MANY of the outstanding advances in the electrical and electronic fields made in the past few years would have been impossible had the plastics industry not been in a position to supply or to develop the various insulating materials necessary. It would be invidious to single out any particular plastic material, since nearly all of them found application, but the vinyls, polyethylene, polystyrene, polybutene various laminates, and polyester resins played major roles. Although all plastics are essentially insulators, or nonconductors of electricity, a very wide range of electrical properties is to be found among them and nowadays it is just as important for the plastics technologist to be familiar with the dissipation factor and dielectric constant of the materials in which he is interested as it is for him to know the heat distortion point, the tensile strength, and the ability to withstand outdoor weathering.

Many articles have been written giving the electrical properties of various proprietary plastics, and from these the technical man can obtain approximate values for such parameters as dissipation factor, arc resistance, and so forth. It is not intended in this discussion that we should make a recapitulation of such data. Rather it is intended to show how the electrical properties of the materials in which we are interested are dependent on a number of factors, what changes can occur or can be made to occur, and how a better understanding of the behavior, mechanical as well as electrical, can perhaps be achieved by the use of electrical test procedures.

The two electrical properties most commonly quoted and measured are the dissipation factor and dielectric constant. The dielectric constant ϵ' is defined as the ratio of the capacitance of a capacitor with the dielectric material placed between the two metallic plates acting as electrodes, to the capacitance of the same arrangement of plates in a vacuum at the same temperature. The choice of the term "constant" in this respect is unfortunate, as the parameter may change quite markedly with the test conditions of frequency and temperature. The loss factor ϵ'' is equal to the product of the dielectric constant and the cosine of the phase angle θ between the voltage impressed on, and the alternating current flowing in, the material under examination placed between suitable metallic electrodes. The ratio ϵ''/ϵ' is the dissipation factor and is commonly referred to as tan δ, where δ is 90 degrees minus θ, whereas the power factor is given strictly by $\sin\delta=\cos\theta$. It can be seen, however, that for small values of ϵ''/ϵ', dissipation factor, tan δ, and power factor are for all practical purposes equal.

It is necessary at this stage to give a little background of the theory of dielectrics although a much clearer and complete picture can be obtained by studying the literature already published. It was early postulated by Maxwell that the square of the refractive index of a substance measured by visible light was equal to the dielectric constant, but Debye in 1912, was the first to give some explanation of the fact that a large number of substances possess dielectric constants greater than those calculable from their refractive indices. He explained that these substances possess molecules in which the electrical charges are unsymmetrically placed and that in an alternating electric field therefore, they tend to align themselves in the direction of the applied field. This separation of the electrical charges in a molecule forms what is termed a "dipole," and making various assumptions, Debye deduced an equation based on this concept:

$$\frac{\epsilon'-1}{\epsilon'+2}\cdot\frac{M}{d}=\frac{4\pi N\alpha}{3}+\frac{4\pi N}{3}\cdot\frac{\mu^2}{3kT}\cdot\left(\frac{1}{1+i\omega\tau}\right),$$

where

M = molecular weight,

d = density,

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N = Avagadro's number $= 6.06 \times 10^{23}$,
α = polarizability of the molecule,
μ = permanent or dipole moment,
k = Boltzmann's constant,
$\omega = 2\pi$ times the frequency at which the measurement was made, and
τ = relaxation time of the dipole.

It is, we believe, apparent that τ, the relaxation time, is proportional to the internal viscosity of the material under examination, and for spherical molecules of radius α_1, the expression

$$\tau = \frac{8\pi\eta\alpha_1^3}{2kT}$$

has been derived, where η = the viscosity, and the other terms are as previously defined.

The expressions so far given have been found to hold strictly only for a limited number of very simple molecules in the gas phase, or for dilute solutions of certain polar molecules in nonpolar solvents, but the use of the Debye equation as a first approximation has yielded many valuable results. Other investigators have attempted to "revise" the Debye equation in an attempt to make the observed results fit the theory, but it must be confessed that we still do not possess an adequate understanding of these phenomena.

The first term on the right-hand side of the general expression gives the polarizability due to the displacement of charges within the atoms themselves. This value is assumed to be inversely proportional to the force binding the charges to the nuclei of the atoms and directly proportional to the number of bound electrons per unit volume. Since these charges can form in times less than 10^{-10} seconds, they usually contribute a fixed amount to the dielectric constant in the electrical frequency range. For strictly nonpolar materials, for example benzene, this is the only form of polarizability present, the dielectric constant is equal to the square of the refractive index and the value is independent of frequency. For such nonpolar materials, the dielectic constant then changes only with density changes as shown by:

$$\frac{\epsilon_\infty - 1}{\epsilon_\infty + 2} \cdot \frac{M}{d} = constant.$$

This affords a novel way of determining electrically any transition point in a nonpolar polymer due to sharp changes in rate of change of density, provided the method employed is sensitive enough to check such differences of dielectric constant. It should therefore be possible to determine, for example, the second-order transition point of polystyrene or to study the density changes of polyethylene over a temperature range by this method.

The second term of the general equation shows that, for polar materials, the dielectric constant is dependent not only on temperature but also on frequency and the viscosity of the medium. Moreover, the theory predicts that there will be a maximum in the dissipation factor, which will occur at the midpoint of the change of dielectric constant from the low value at very high frequencies to the so-called static dielectric constant or the value measured by direct current. We are not proposing to deal here with the derivation of this point of maximum dissipation factor nor to show how theory fails to account for all of the observed phenomena but wish only to show how this maximum can be used to assist us in several of our plastics problems.

In addition to the dissipation factor caused by the polar nature of the molecules under investigation we often get an additional electrical loss at elevated temperatures due to the contributions

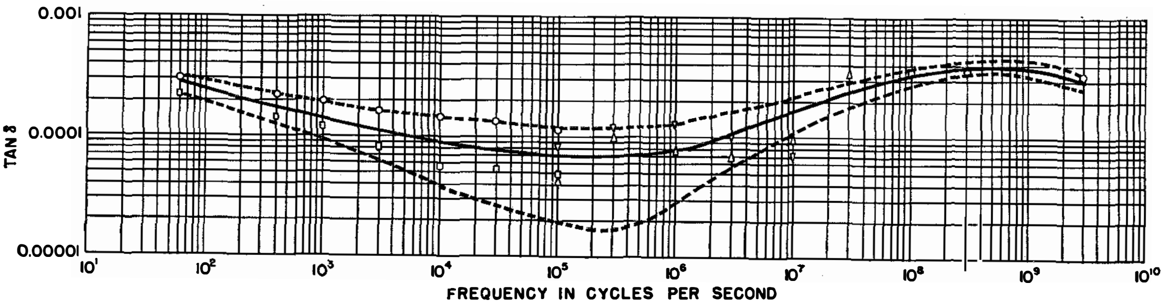


Fig. 1—Polvtetrafluoroethylene at 24 degrees centigrade.

of what we may term "adventitious" ionic impurities, and in many commercial plastics it is often this ionic loss which determines the usefulness of the material for electrical applications. The volume resistivity of a commercial plastic is often dependent also on this "adventitious" ionic material as will be seen later.

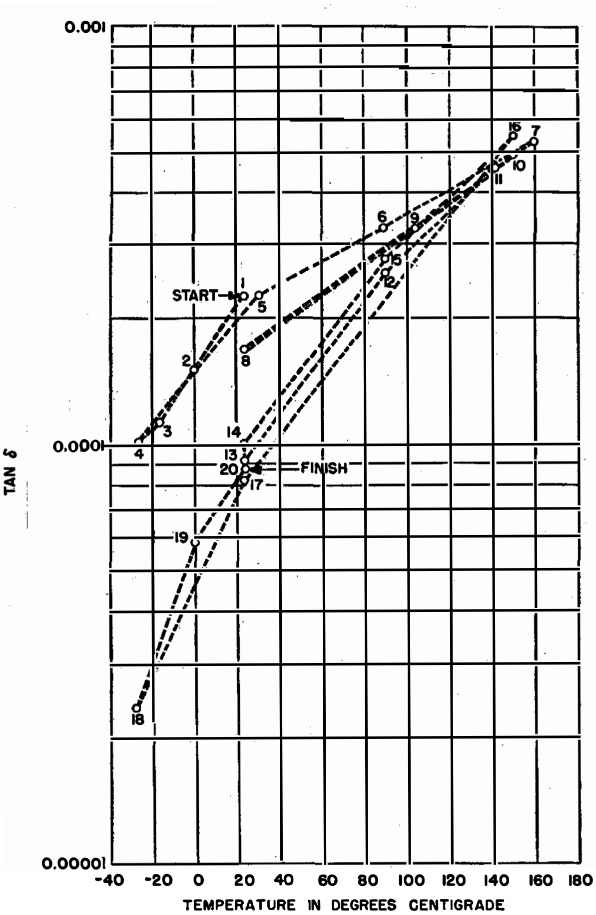


Fig. 2—Polytetrafluoroethylene at 60 cycles per second.

To illustrate the various points under discussion, we have chosen various examples from our experience that we believe to be of general interest. First, let us examine a nonpolar polymer, and for this we would use the one with the lowest electrical losses we have so far measured, namely polytetrafluoroethylene. From an examination of the X-ray and other data for this material, we get a picture of a highly compact molecule comprising a rod of carbon atoms surrounded by attached fluorine atoms. The chain packing is tight, but since the molecule is essentially rod-like, it is possible to envisage a degree of freedom not normally found in high-polymeric materials, namely, rotation round the axis of the molecule. Fig. 1 shows the actual results of measurements of dissipation factor made on this material at a constant temperature of 24 degrees centigrade and over the frequency range of 60 to 3×10^9 cycles per second. It will be seen that this material has a very low value of dissipation factor and that there is evidence of a "peak" in the 5×10^8-cycle-per-second range and a slight rise in value as the frequency decreases from 10^5 cycles per second. It is difficult on the basis of existing knowledge to determine the cause of the "peak" observed, although its position would perhaps indicate the presence of traces of polar material. Since this material is known to be more sintered than fused, it is possible that the low-frequency losses may be due to interfacial phenomena, which are outside the scope of this present discussion. In Fig. 1, the center curve is the most probable value for the material from determinations on many samples, while the upper and lower curves give the maximum and minimum values, respectively, of the various samples.

Another interesting use of electrical measurements on this material is shown in Fig. 2 in which we plot dissipation factor at 60 cycles per second against temperature with heat cycling of a sample of sheet polytetrafluoroethylene. Heat cycling was performed from -30 to 160 degrees centigrade and it will be observed that cycling causes a progressive decrease in the lower-temperature values indicating again that the material is not electrically isotropic. Since this material is intended for use in electrical equipment over a very wide range of temperatures, it is interesting to note the relatively large coefficient of dissipation factor with temperature.

Another interesting application of electrical measurements on nonpolar materials arose during the recent war in connection with radar cables. It was observed that certain equipments lost their sensitivity with time, and this loss was traced back to the high-frequency cables employed. For these applications, the dielectric used was polyethylene which has a dissipation factor of around 0.00035 at 3×10^9 cycles per second (in the operating range of the equipment). However, on aging, the dissipation factor was found to increase, especially when the cable was operating at elevated temperatures. Polyethylene by itself does not show this characteristic, and the increase was finally traced down to migration of plasticizer (usually di-2-ethylhexyl phthalate) from the vinyl jacket to the polyethylene. In finding a solution of the problem by making the so-called "noncontaminating" jacket, the use of dissipation factor measurements was the only means of rapidly detecting the migration of plasticizers from jacket stock. Fig. 3 shows the kind of results obtained with polyethylene and tricresyl phosphate and di-2-ethylhexyl phthalate at 85 degrees centigrade and also for comparison, a result obtained with polytetrafluoroethylene and di-2-ethylhexyl phthalate at 200 degrees centigrade.

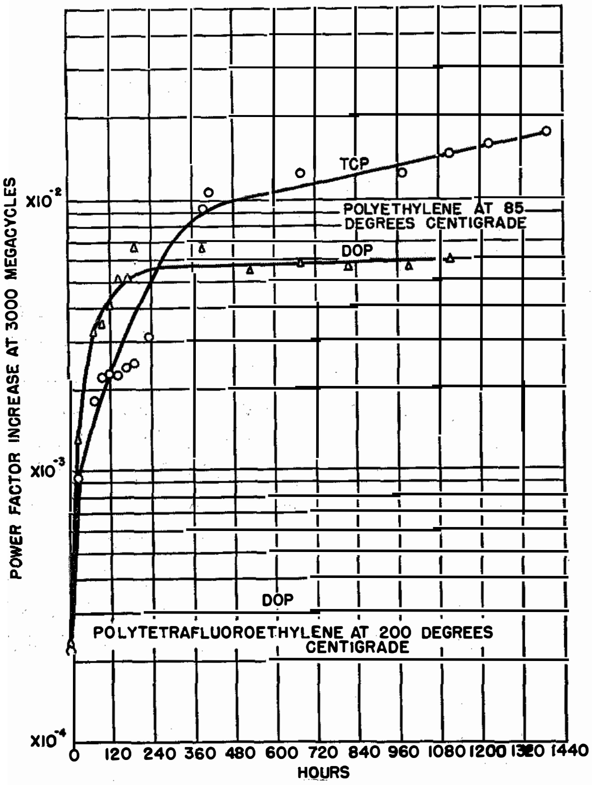


Fig. 3—Effect of plasticizers on electrical properties.

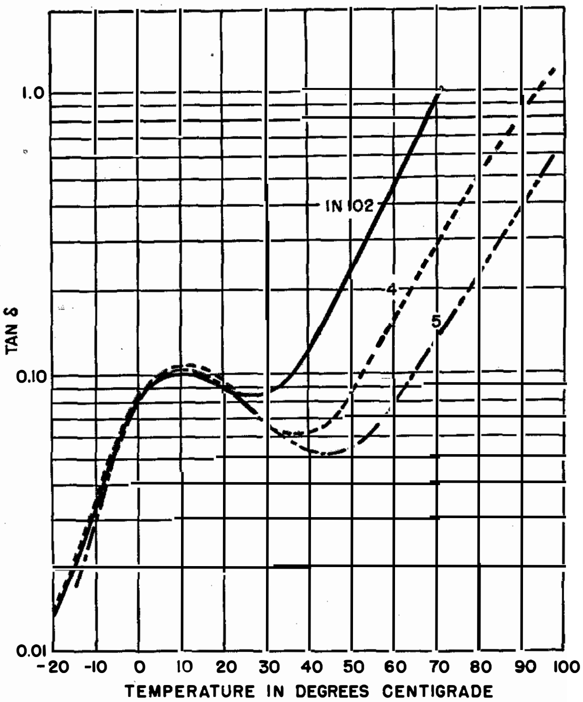


Fig. 5—Tangent δ versus temperature at 60 cycles.

Coming now to polar materials, it is interesting to study the vinyl compounds, especially those used as primary insulation or as jackets for electrical wires and cables. A study of the dissipation factor and dielectric constant of such a material is exemplified by Fig. 4, which shows the results obtained on the so-called "noncontaminating" jacket, IN-102. It will first be observed that the dissipation factor is some two and one half decades greater than a nonpolar material such as polystyrene or polyethylene and also that there is a pronounced broad peak whose maximum occurs at about 10^4 to 10^5 cycles per second. The dielectric constant decreases from a low-frequency

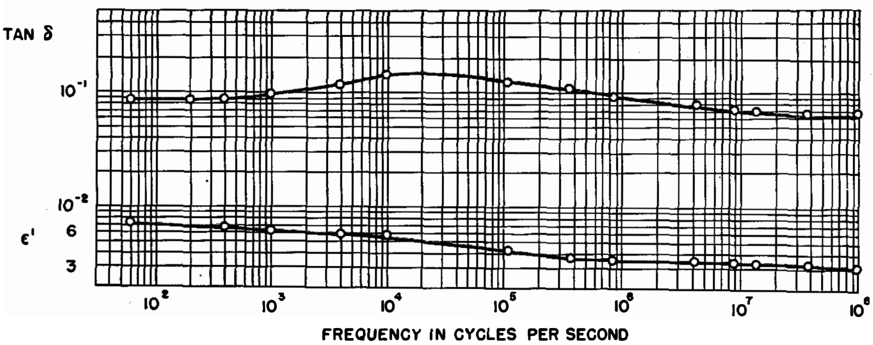


Fig. 4—IN-102 at 30 degrees centigrade.

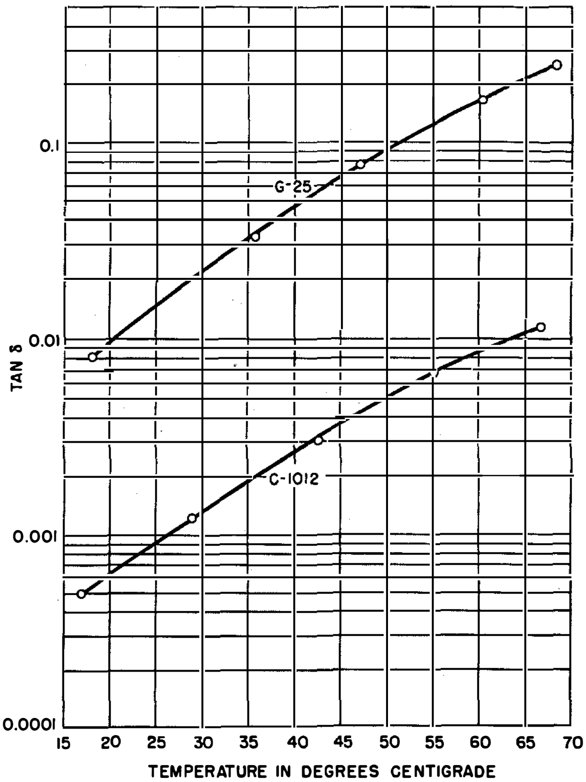


Fig. 6—Tangent δ versus temperature at 0.4 kilocycle.

value of 7 to a value at a frequency of 10^7 cycles per second of 3. It is not usual, however, to employ this type of material at high frequencies, and hence a knowledge of the low-frequency behavior, especially over a temperature range, is important. Fig. 5, therefore, shows the results obtained on the same compound at 60 cycles per second over a temperature range of -20 to 100 degrees centigrade. It will be observed that on going from the low temperature to a high temperature, the dissipation factor first rises, then falls, and then rises again. The first rise, giving a peak at 12 degrees centigrade, is due to the dipolar contribution to the loss, while the second rise, which reaches a constant rate of rise, is due to ionic materials present in the compound. It is obviously desirable to keep the ionic losses to a minimum since, as is shown in the curve, these are the chief losses at temperatures above 50 degrees centigrade, and therefore they tend to limit the operating temperature. In curve *4* of Fig. 5, we give the results on a compound in which we have changed the stabilizer employed to one that we have found to be more effective in preventing decomposition of the vinyl chloride acetate resin, but otherwise the composition is the same as the regular IN-102. It will be observed that the dipolar contribution to the loss is unchanged but that a marked improvement of the loss at higher temperatures has been achieved. Another possible source of ionic loss was felt to be the possibility of residual material in the plasticizer used, a resinous polyester (G-25). Initial experiments indicated the validity of this assumption and a special batch of plasticizer was prepared, paying particular attention to keeping the ionic impurities to a minimum (C-1012).

Fig. 6 shows the results obtained by measuring the dissipation factor of the original (G-25) and the purified (C-1012) material at 4×10^2 cycles per second against temperature, and it will be observed that whereas the results on the purified material run parallel to those of the original material, they are in general slightly more than one decade better.

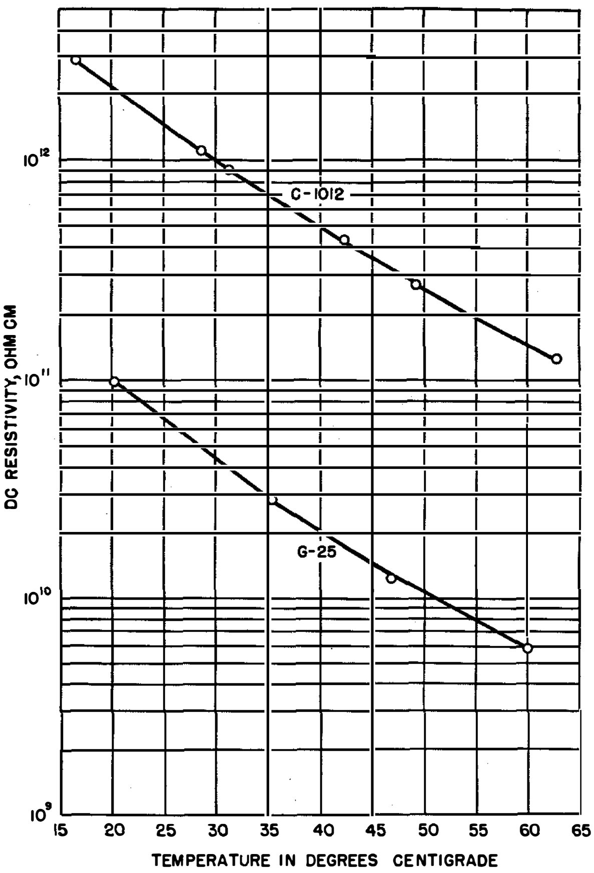
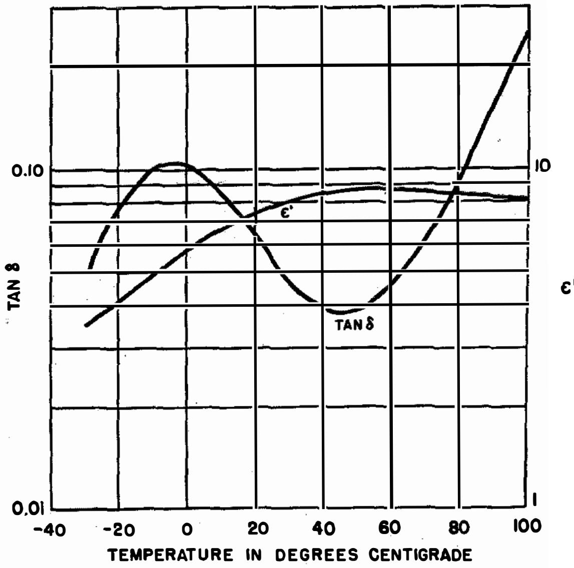


Fig. 7—Direct-current resistivity versus temperature.

Fig. 7 shows the corresponding results for measurements of direct-current resistivity, the two materials again giving parallel curves with the purified material being just over one decade superior. This indicates that the improvement in dissipation factor has been caused by the removal of ionic impurities. Curve *5* of Fig. 5 shows the results obtained when the improved stabilizer and the improved plasticizer are used, and the resulting compound is seen to have considerably improved electrical properties at elevated temperatures over the original IN-102. We believe that these results indicate a very neat approach to the problem of testing plasticizers and stabilizers in vinyl compounds of this type.

Another problem which is very interesting both technically and technologically is that of the low-temperature performance of various semirigid plastics and the attempt to determine the behavior of such materials when applied in some fabricated form such as the extruded jacket of a cable. We are all familiar with the brittle point (A.S.T.M. Method D 746[1]) and the torsion stiffness tester (Berg and Clash),[2] but most technologists recognize that sometimes the results obtained by these tests vary from those obtained in actual performance tests. Moreover, such tests have always to be made on a compounded stock, and it is therefore not easy to study more fundamental questions of plasticizer efficiency by such tests. Some preliminary work on the possible utilization of an electrical measurement approach to this problem has been made by other investigators such as Leilich and Würstlin, who have shown in rubber and allied compounds that the more flexible a material is at low temperatures the lower is the temperature of the maximum of the dissipation factor curve at a given frequency. We have made a preliminary study of vinyl compounds and find that there is a correlation between the temperature of the dissipation factor maximum and the so-called "brittle-point." Fig. 8, for example, shows the results obtained on two vinyl compounds whose dissipation factor maxima occur at 20 degrees centigrade and -2 degrees centigrade, respectively. IN-104, with a maximum temperature of 20 degrees centigrade, is a primary insulation comprising polyvinyl chloride-acetate plasticized with dioctyl phthalate and tricresyl phosphate, whereas IN-105, with a maximum temperature of -2 degrees centigrade, is a jacketing composition comprising the same base resin but using only dioctyl phthalate as plasticizer and in larger amounts than the IN-104. The brittle points for the two compounds are -18 and -42 degrees centigrade, respec-



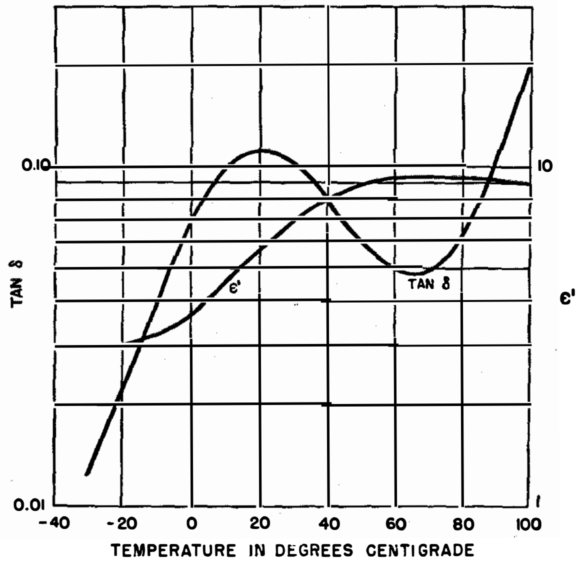


Fig. 8—ϵ' and tangent δ versus temperature at 60 cycles. (Upper) Compound IN-105 black. (Lower) Compound IN-104 cream.

[1] Tentative Method of Test for Brittle Temperature of Plastics and Elastomers (D 746–44 T) 1946 Book of A.S.T.M. Standards, Part III-B; p. 817.

[2] R. M. Berg and R. F. Clash, Jr., "Vinyl Elastomers Low-Temperature Flexibility Behavior," *Industrial and Engineering Chemistry*, v. 34, p. 1218; 1942.

tively, and it can be seen that the difference between the brittle-point values is of the same order as the difference between the temperature of the dissipation-factor maxima. We have also made measurements of the temperature of the dissipation-factor maximum of various plasticizers and the values for the same parameter measured on compounds using these plasticizers, and believe that from such data we can predict not only the low-temperature brittle points of the compounds, but also get some understanding of the efficiency of various materials as plasticizers. This phase of the work is under active study in our laboratories at the present time.

It is recognized that this discussion affords only a brief and very cursory examination of the problem of the electrical properties of plastic materials, but it is to be hoped that we have shown that electrical test methods can be effectively used in the further development of improved materials.

Discussion

MR. GEORGE E. POWER.[3]—Am I correct in the assumption that the maximum loss factor as determined on a loss-factor-versus-temperature curve is the same as the maximum determined on a loss-factor-versus-frequency curve?

MR. A. J. WARNER (*author*).—That is essentially correct. Although the absolute values may be slightly different, the order of magnitude is the same. It is also possible to determine the frequency at which the maximum loss factor will occur from the loss-factor-versus-temperature curve.

MR. POWER.—Does the frequency at which the loss-factor-versus-temperature curve is determined have any effect on the maximum loss factor?

MR. WARNER.—It does not affect the order of magnitude of the maximum but it will affect its position on the curve. We have done most of our work at 60 cycles.

MR. R. L. STERN.[4]—I wonder whether Mr. Warner is familiar with a recent article,[5] "The Dielectric Identity Test for Plasticizers," which appeared in *Analytical Chemistry*, for January, 1947, and whether it could be cited here?

[3] Engineering Department, The Formica Insulation Company, Cincinnati, Ohio.

[4] Chemical Superintendent, Hercules Powder Company, Parlin, New Jersey.

[5] By M. A. Elliott, A. R. Jones, and L. B. Lockhart.

Antennas for Circular Polarization*

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A FORMULA is derived to give the variation in received voltage when an elliptically polarized antenna is rotated in a plane transverse to the direction of propagation of the incident elliptically polarized wave. It is shown that a circularly polarized antenna will not receive any of its transmitted energy that is reflected from a highly conducting smooth surface. Conditions that must be satisfied to obtain an omnidirectional circularly polarized pattern are derived. Experimental results are given.

• • •

1. Introduction

The aim of this paper is to examine some aspects of circularly polarized antennas, and more generally the case of elliptically polarized systems. If a system of coordinates is chosen as shown in Fig. 1, the electric field of an elliptically polarized plane wave traveling in the positive z direction is given by

$$\boldsymbol{E} = \iota_x \mathcal{E} \exp [j(\omega t - \beta Z)] + \iota_y \mathcal{E} \exp [j(\omega t - \beta Z + \theta)], \quad (1)$$

where

ι_x and ι_y = unit vectors in x and y directions
$\beta = 2\pi/\lambda$
λ = wavelength
θ = phase difference between x and y components.

The actual field varies as the real part of this expression, each component being of unit amplitude. If $\theta = \pm\pi/2$, circular polarization is obtained. If the sign of $\pi/2$ is positive, the field is said to be right-handed circularly polarized; if the sign is negative, the field is left-handed circularly polarized. Fig. 1 shows the sense of rotation of a right-handed circularly or elliptically polarized field at a fixed instant of time. Viewed from the origin, the electric field vector rotates clockwise around the z axis in the direction of propagation (in the same sense as a nut rotating on a right-handed screw thread). At a fixed value of z, however, the field vector rotates counterclockwise around the z axis as a function of time, looking again in the direction of propagation. The essential difference between elliptical and circular polarization is that the amplitude of the field vector for elliptical polarization varies in such a way that it describes an ellipse as it rotates about the z axis, whereas for circular polarization the amplitude remains constant. A circularly polarized field that is right-handed with respect to space is left-handed with respect to time. Confusion is avoided by specifying the sense in only one coordinate, the space coordinate.

An expression is derived in the Appendix which gives the absolute magnitude of voltage V induced in an elliptically polarized receiving antenna located in the field of an elliptically polarized wave. The expression is

$$V = K\left(1 \mp \frac{2r_2}{r_2^2+1}\,\frac{2r_1}{r_1^2+1} + \frac{r_2^2-1}{r_2^2+1}\,\frac{r_1^2-1}{r_1^2+1}\cos 2\alpha\right)^{\frac{1}{2}}, \quad (2)$$

where

K = a constant
r_1 = ratio of maximum to minimum amplitudes in the receiving antenna as it is rotated in a plane transverse to the direction of propagation of a linearly polarized plane wave. It is also the ratio of maximum to minimum amplitudes of the field vector in the elliptically polarized wave that would be radiated by the receiving antenna if it acted as a transmitter
r_2 = ratio of maximum to minimum amplitudes of the field vector of the incident elliptically polarized wave
α = angle between the direction of maximum amplitude of the electric-field vector in the incident elliptically polarized wave and the direction of maximum amplitude of the

* Reprinted from *Proceedings of the I.R.E.*, v. 36, pp. 997–1001; August, 1948. Presented, Institute of Radio Engineers National Convention, March 6, 1947, New York, New York.

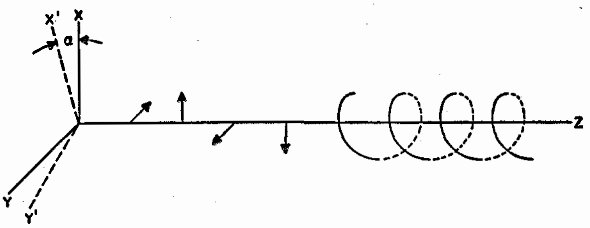

Fig. 1—Electric field of an elliptically polarized wave traveling from left to right.

> electric-field vector that would be produced by the receiving antenna if it were radiating. It is assumed that the latter electric-field vector always lies in a plane normal to the direction of propagation of the incident wave.

The $\mp$ sign will be read as $+$ if both the receiving antenna and the transmitting antenna produce the same handedness or screw sense of polarization, and as $-$ if they have opposing senses of polarization.

2. *Examples*

Let us now consider two particular cases.

2.1 Case A

A circularly polarized wave is incident on a circularly polarized receiving antenna.

Then $r_1 = r_2 = 1$, and (2) becomes

$$\left.\begin{aligned} V &= K(1\mp 1)^{\frac{1}{2}} \\ &= K\ 2^{\frac{1}{2}} \text{ or } 0 \end{aligned}\right\}. \tag{3}$$

Thus, the receiving antenna will absorb energy if it has the same handedness of polarization as the incident wave. If the antenna has the opposite screw sense to that of the incident wave, it will be completely "blind" to such radiation and will absorb no power. This holds for all angles around the z axis in the xy plane, Fig. 1, at which the receiving antenna can be inclined, (3) being independent of α. One conclusion is that, if circular polarization is to be used for communication, the transmitting and receiving antennas must produce the same handedness of polarization when each is used as a radiator.

One can go further and predict the behavior of a circularly polarized antenna as a receiver of energy reflected back to it from a highly conducting smooth surface. This system has the property that, if the wave incident on the reflector is right-handed, the reflected wave will be left-handed, since both of the mutually perpendicular components of electric field in the incident wave are shifted 180 degrees in phase on reflection, but the direction of propagation in the reflected wave is reversed. This analysis does not apply to metallic reflectors made of parallel thin wires, because such surfaces will reflect mainly the component of polarization parallel to the wires. The fact that a circularly polarized antenna will not receive energy reflected from a metallic object can be useful in many ways; for instance, the impedance of a circularly polarized antenna does not change when it is used to excite a parabola. During the war, considerable trouble was encountered in matching radiators associated with full paraboloids having focal lengths a few wavelengths long, even over relatively narrow frequency bands, because of the reflection introduced into the transmission line by the paraboloid. The voltage reflection coefficient in such a case is given[1] by

$$\Gamma = \frac{G\lambda}{4\pi F}, \tag{4}$$

where

Γ = voltage reflection coefficient
G = gain of antenna in direction of vertex (not the gain of the whole antenna)
λ = wavelength
F = focal length.

This formula was derived for linear polarization. For elliptical polarization, by use of (2), (4) becomes

$$\Gamma = \frac{G\lambda}{4\pi F}\,\frac{r^2-1}{r^2+1}, \tag{5}$$

where r has the same meaning as r_1 in (2).

The reflection coefficient at 2200 megacycles per second due to a paraboloid having a 24-inch diameter by 9-inch focal length was measured with an antenna that could be changed to produce circular, elliptical, or linear polarization. For each polarization, the antenna was matched with a double-stub tuner to give a standing-wave ratio less than 1.05 in free space. The antenna

[1] S. Silver, Report No. 422, Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass.

was then moved back and forth on a line between the vertex and the focus, and the maximum and minimum standing-wave ratios were measured. From these measurements, the reflection coefficients were obtained for the case where the antenna and paraboloid reflections add, and for the case where one subtracts from the other. From these two reflection coefficients, the true reflection is obtained. Table 1 summarizes the data.

TABLE 1

MATCHING BETWEEN ANTENNA AND PARABOLOID

Polarization	Reflection Coefficient	
	Calculated	Observed
Circular	0.00	0.01
Elliptical	0.06	0.07
Linear	0.08	0.10

2.2 Case B

Consider now an elliptically polarized wave incident on an elliptically polarized antenna, so that r_1 is not equal to r_2. Inspection of (2) shows that the power received will depend on the angle α at which the receiving antenna is oriented. Also, more energy will be absorbed when the screw senses of polarization are the same than when they are not, although there will be no orientation for which the received power will be zero.

TABLE 2

RECEIVED VOLTAGE BETWEEN ELLIPTICALLY POLARIZED ANTENNAS

	Same Sense of Polarization		Opposite Sense of Polarization	
	Maximum Voltage	Minimum Voltage	Maximum Voltage	Minimum Voltage
Calculated	1	0.775	0.638	0.040
Observed	1	0.708	0.600	0.037

Table 2 gives confirming experimental data. Measurements were made at 1000 megacycles, using two turnstile antennas so that the screw sense could be easily changed. The transmitting antenna was placed about four wavelengths away from the receiving antenna. The receiving antenna was rotated in the plane transverse to the direction of propagation (xy plane, Fig. 2), and the maximum and minimum received powers were measured. One dipole was then rotated 180 degrees to change the screw sense (i.e., in Fig. 2, connections to arms B and B' were reversed while leaving the connections to dipole AA' unchanged), and maximum and minimum received powers were again measured. Ratios r_1 and r_2 were measured by replacing one of the antennas with a dipole and measuring the maximum and minimum powers received as the dipole was rotated. All values are normalized with respect to the greatest value of induced voltage; values of ratios r_1 and r_2 were 2.0 and 2.24, respectively.

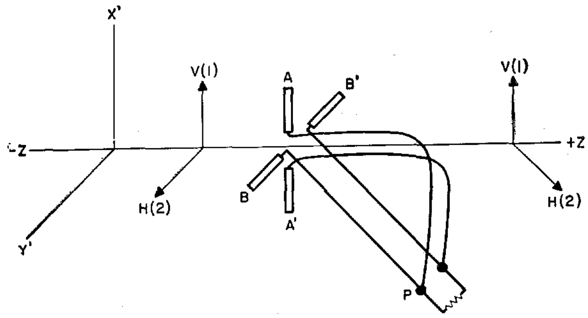

Fig. 2—Use of turnstile antenna to demonstrate effect of screw sense of polarization between elliptically polarized wave and the receiving antenna.

A circularly polarized antenna can, of course, be used to receive a linearly polarized wave, or a linearly polarized antenna to receive power from a circularly polarized wave.

3. *Methods of Producing Circular Polarization*

A simple circularly polarized antenna with a field pattern similar to that of a dipole has been described.[2]

The antenna consists of a horizontal loop with a vertical dipole at its center placed normal to the plane of the loop, Fig. 3. Although actual loops are built in the form of triangles or squares, the loop will be considered to be a circle for this discussion. In the plane of the loop, the field dE_h from two diametrically opposite elements $ad\theta$ on

[2] A. G. Kandoian, "Three New Antenna Types and Their Applications," *Proceedings of the I.R.E.*, v. 34, pp. 70W–75W; February, 1946. Also, *Electrical Communication.*, v. 23, pp. 27–34; March, 1946.

the periphery of the loop[3] is

$$\left.\begin{aligned} dE_h &= C_1\{\mathcal{E}\exp[j(\omega t - k_1 b - kr + ka\cos\theta)] \\ &\quad - \mathcal{E}\exp[j(\omega t - k_1 b - kr - ka\cos\theta)]\} \\ &\qquad\qquad \times\cos\theta d\theta \\ &= 2jC_1\mathcal{E}\exp[j(\omega t - k_1 b - kr)] \\ &\qquad\qquad \times\sin(ka\cos\theta)\cos\theta d\theta, \end{aligned}\right\} \quad (6)$$

where

C_1 = constant
$k_1 = 2\pi/\lambda_1$
λ_1 = wavelength in transmission line
$k = 2\pi/\lambda$
λ = free-space wavelength
b = length of transmission line
a = radius of loop
r = distance from center of loop.

The loop is considered to be made of infinitesimal elements, each excited uniformly by a separate transmission line. The total field is obtained by integrating (6) from 0 to π.

$$E_h = jC_2\mathcal{E}\exp[j(\omega t - k_1 b - kr)]J_1(ka), \quad (7)$$

where J_1 = Bessel function of the first order. Equation (7) shows that the phase of the distant field, referred to the center of the loop, has undergone a shift of a quarter-wavelength, which is

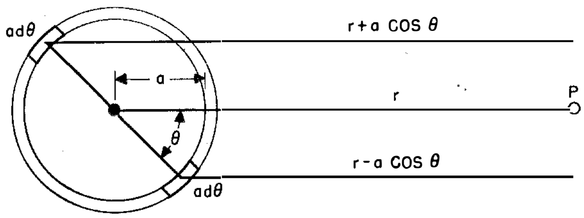

Fig. 3—Production of a circularly polarized wave by means of a horizontal loop antenna with a vertical dipole placed at the center.

independent of the transmission-line length b, or the distance r. The distant field of the vertical dipole at the center of the loop is

$$E_v = C_3\mathcal{E}\exp[j(\omega t - kr)]. \quad (8)$$

The factors C_2 and C_3 in (7) and (8) contain all parameters that affect the magnitude but not the phase of the distant field. To obtain circular polarization, two conditions must be satisfied:

$$C_2 J_1(ka) = C_3,$$

and

$$\mathcal{E}\exp[-jk_1 b] = \pm 1.$$

The first condition must be satisfied to produce equal horizontal and vertical fields. The loop diameter must be less than about 0.6 wavelength to obtain a pattern that is roughly equivalent to that from a vertical dipole.[3] The second condition must be satisfied if the phase of the horizontal field is to differ by a quarter wavelength from the vertical field. This condition is satisfied by making length b of the transmission line connected to the loop elements an integral number of half wavelengths long. Another way of stating this is that the current in the loop must be exactly in phase or exactly out of phase with the current in the dipole.

There are certain types of circularly polarized antennas that produce both right-handed and left-handed circular polarization simultaneously in different directions. The antenna shown in Fig. 2, the well-known turnstile,[4] which is used mainly to obtain a horizontally polarized omnidirectional pattern, is one of these. In Fig. 2 is shown the orientation of the field at one instant of time and a quarter cycle later. At time l, the field at two points on the z axis on opposite sides of, and equidistant from, the turnstile is vertical; the horizontal component is zero. A quarter cycle later the field is horizontal. The field traveling along the plus-z axis rotates clockwise in time, or to the left hand in space. The field traveling along the minus-z axis rotates counterclockwise in time, or right-handedly in space. This means that, with a 180-degree rotation of the turnstile antenna about an axis transverse to the direction of propagation, the received power will go from a maximum to zero when received on a circularly polarized fixed receiving antenna.

A turnstile antenna can be made more directive by placing a reflector a quarter wavelength behind it, because the reflected field will then have the same screw sense as the direct field. A plane reflector cannot be used with an antenna like that described in footnote reference 2 (which produces the same screw sense in all directions) because the reflected field will have the opposite screw sense from the direct field.

[3] D. Foster, "Loop Antennas with Uniform Current," *Proceedings of the I.R.E.*, v. 32, pp. 603–607; October, 1944.

[4] G. H. Brown, "The Turnstile Antenna," *Electronics*, v. 9, p. 15; April, 1936.

4. Appendix

Consider the second system of coordinates, x', y', and z' obtained from those of Fig. 1 by a rotation about the z axis through an angle α.

Referred to this system, the value of $\boldsymbol{E}$ given by (1) becomes

$$\left.\begin{aligned}\boldsymbol{E} &= \mathbf{i}_{x'}\{\mathcal{E}\exp[j(\omega t-\beta Z)]\cos\alpha+\mathcal{E}\exp[j(\omega t-\beta Z+\theta)]\sin\alpha\} \\ &\qquad +\mathbf{i}_{y'}\{-\mathcal{E}\exp[j(\omega t-\beta Z)]\sin\alpha+\mathcal{E}\exp[j(\omega t-\beta Z+\theta)]\cos\alpha\} \\ &= \mathbf{i}_{x'}(1+\cos\theta\sin 2\alpha)^{\frac{1}{2}} \\ &\quad\cdot\mathcal{E}\exp\left\{j\left[\omega t+\arctan\left(\frac{\sin\theta\sin\alpha}{\cos\alpha+\cos\theta\sin\alpha}\right)-\beta Z\right]\right\}+\mathbf{i}_{y'}(1-\cos\theta\sin 2\alpha)^{\frac{1}{2}} \\ &\quad\cdot\mathcal{E}\exp\left\{j\left[\omega t+\arctan\left(\frac{\sin\theta\cos\alpha}{-\sin\alpha+\cos\theta\cos\alpha}\right)-\beta Z\right]\right\} \\ &= \mathbf{i}_{x'}E_A+\mathbf{i}_{y'}E_B.\end{aligned}\right| \quad (9)$$

In general, the difference in phase between the x' and y' components of the field will depend on the value chosen for α. When $\alpha=45$ degrees, (9) assumes the especially simple form

$$\boldsymbol{E}=\mathbf{i}_{x'}(1+\cos\theta)^{\frac{1}{2}}\mathcal{E}\exp[j(\omega t-\beta Z+\theta/2)]+\mathbf{i}_{y'}(1-\cos\theta)^{\frac{1}{2}}\mathcal{E}\exp[j(\omega t-\beta Z+\theta/2-90°)]. \quad (10)$$

Hence, we see that any wave of the form given by (1) or (9) is equivalent to one of the form (10), where we have two mutually perpendicular components in space of amplitudes $(1+\cos\theta)^{\frac{1}{2}}$ and $(1-\cos\theta)^{\frac{1}{2}}$ and in phase quadrature.

Now suppose a receiving antenna placed at the point $z=z_0$ on the z axis, consisting of two mutually perpendicular half-wave dipoles AA' and BB', Fig. 2, both parallel to the xy plane, and with their axes coinciding with those of the x' and y' axes.

Assume also that dipole A is fed by a line that is Δl longer or shorter than the line supplying power to dipole B. If the sign of Δl is $+$, this antenna of Fig. 2 is right-handed elliptically polarized in the negative-z direction. If the sign is $-$, the antenna is left-handed elliptically polarized in the negative-z direction.

When the wave described by (9) reaches this antenna, voltages will be induced in dipoles A and B as a result of the x' and y' components of $\boldsymbol{E}$, respectively. Assuming that mutual-impedance effects are zero, voltages V_A and V_B induced by dipoles A and B, respectively, at the common junction are

$$V_A=k_AE_A\mathcal{E}\exp[-j\beta(1\pm\Delta l)], \qquad V_B=k_BE_B\mathcal{E}\exp[-j\beta 1], \quad (11)$$

where k_A and k_B are proportionality constants, which, in general, differ from one another.

At point P, Fig. 2, the equivalent circuit is as in Fig. 4. Here, Z_A and Z_B are the impedances at point P looking into the lines leading to antennas A and B, respectively. By Thevenin's theorem, we obtain for the voltage drop V across the load,

$$V=\frac{(V_AZ_B+V_BZ_A)}{Z_L(Z_A+Z_B)+Z_AZ_B}Z_L=\frac{\{k_AE_AZ_B\mathcal{E}\exp[-j\beta(1\pm\Delta l)]+k_BE_BZ_A\mathcal{E}\exp[-j\beta 1]\}}{Z_L(Z_A+Z_B)+Z_AZ_B}Z_L, \quad (12)$$

when relations (11) are used.

One special case that covers a variety of situations is considered. Assume that both dipoles have the same input impedance and that both are matched to their respective transmission lines. Also assume that the load impedance Z_L is real. Then,

$$k_A=k_B=k, \qquad Z_A=Z_B=R, \qquad Z_L=R_L.$$

Equation (12) then takes the following form, after substituting for E_A and E_B from (9):

$$V=\frac{k}{2+R/R_L}\mathcal{E}\exp\left[j(\omega t-\beta Z-\beta 1)\right]\Big\{(1+\cos\theta\sin 2\alpha)^{\frac{1}{2}}$$
$$\cdot\mathcal{E}\exp\left\{j\left[\arc\tan\left(\frac{\sin\theta\sin\alpha}{\cos\alpha+\cos\theta\sin\alpha}\right)\mp\beta\Delta l\right]\right\}+(1-\cos\theta\sin 2\alpha)^{\frac{1}{2}}$$
$$\cdot\mathcal{E}\exp\left\{j\left[\arc\tan\left(\frac{\sin\theta\cos\alpha}{-\sin\alpha+\cos\theta\cos\alpha}\right)\right]\right\}\Big\}, \tag{13}$$

and the absolute value of this complex quantity is

$$|V|=\frac{k}{2+R/R_L}\left[1\mp\sin(\beta\Delta l)\sin\theta+\cos(\beta\Delta l)\cos\theta\cos 2\alpha\right]^{\frac{1}{2}}. \tag{14}$$

Further defining the ratio r_2 as equal to the maximum amplitude of the field vector divided by the minimum amplitude in our elliptically polarized plane wave, we have, from (10),

$$r_2=\frac{(1+\cos\theta)^{\frac{1}{2}}}{(1-\cos\theta)^{\frac{1}{2}}},$$

whence

$$\left.\begin{aligned}\sin\theta&=\frac{2r_2}{r_2^2+1}\\ \cos\theta&=\frac{r_2^2-1}{r_2^2+1}\end{aligned}\right\}. \tag{15}$$

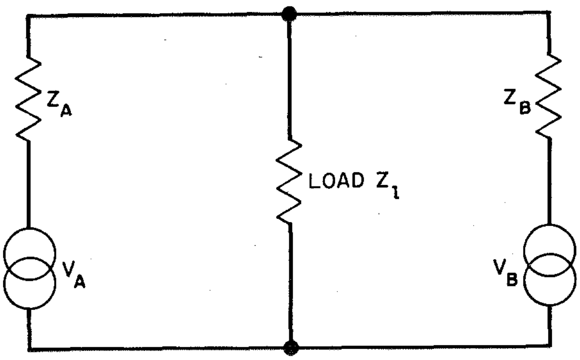

Fig. 4—Equivalent circuit at point *P* of Fig. 2

Similar expressions for the quantities sin $(\beta\Delta l)$ and cos $(\beta\Delta l)$, which appear in (14), can be obtained. For, if the receiving antenna were made to transmit, it would radiate an elliptically polarized wave such that the phase difference between the component due to dipole AA' and that due to dipole BB' would be $\beta\Delta l$. So that, exactly as above, if r_1 is defined as equal to the ratio of maximum-to-minimum amplitudes in the elliptically polarized wave that the receiving antenna could radiate, we have

$$\left.\begin{aligned}\sin\beta\Delta l&=\frac{2r_1}{r_1^2+1}\\ \cos\beta\Delta l&=\frac{r_1^2-1}{r_2^2+1}\end{aligned}\right\}. \tag{16}$$

Then, (14) becomes

$$V=K\left(1\mp\frac{2r_2}{r_2^2+1}\,\frac{2r_1}{r_1^2+1}+\frac{r_2^2-1}{r_2^2+1}\,\frac{r_1^2-1}{r_1^2+1}\cos 2\alpha\right)^{\frac{1}{2}}, \tag{17}$$

which is (2).

Noise-Suppression Characteristics of Pulse-Time Modulation*

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AN EXPERIMENTAL investigation of the noise-suppression characteristics of pulse-time modulation is outlined. Impulse noise and thermal-agitation or fluctuation noise are treated. The effects of these types of noise and the improvements obtained through the use of limiters, differentiators, and multivibrators are presented graphically.

• • •

Communication systems utilizing pulse-time modulation and the general properties of this type of modulation have been described in the technical literature.[1-5] Briefly, in this method, instantaneous samples of the modulating wave vary the time of occurrence of a pulse subcarrier. Thus, a particular value or sample of the modulating signal is represented by the displacement of the pulse in time with respect to a synchronizing pulse or time reference, and the frequency of the modulating signal is given by the rate of change of pulse displacement.

One of the important characteristics of pulse-time modulation is its noise-reducing properties. The noise can be of two distinct types: impulse noise, and thermal-agitation or fluctuation noise. The first may be short impulses caused by electrical disturbances or they may originate in neighboring systems. Thermal-agitation and other noises that have a similar spectral distribution such as "shot" noise are usually contributed by the first few stages of the receiving equipment and, to a lesser degree, by the transmitter if "jitter" exists.[6,7] In general, thermal-noise effects are most important because they determine the lower limit of sensitivity and, hence, such transmission parameters as bandwidth and power.

In a pulse-time system in which the transmitted intelligence is derived from the timing of a pulse edge, noise may displace the pulse edge from the value corresponding to the modulating signal. Noise impulses also may modulate other characteristics of the signal pulses such as amplitude, width, and slope of the pulse edges, but are ultimately translated into pulse-time displacement. The optimum signal-to-noise ratio

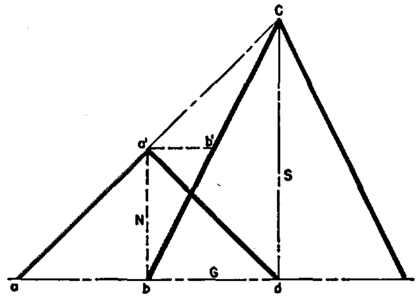

Fig. 1—*S* and *N* are signal and noise impulses. Output noise = $a'b'$. (S/N) output = $D/a'b'$, where D = modulation displacement. $ab = bd = G$. $a'b'/ab = a'b'/G = (N/S)$ input. (S/N) output $= (S/N)$ input D/G.

is realized when all effects of noise, other than time displacement, are eliminated by suppression devices in the receiver.

The method by which noise distorts the signal pulses and causes a distortion of the pulse edge timing is shown in Fig. 1. It should be noted that only direct-current or "video" pulses are treated; they are considered independently of the method of transmission. Where amplitude modulation

* Reprinted from *Proceedings of the I.R.E.*, v. 36, pp. 446–450; April, 1948. Presented, 1947 Institute of Radio Engineers National Convention, March 6, 1947, New York, New York.

[1] E. M. Deloraine and E. Labin, "Pulse-Time Modulation," *Electrical Communication*, v. 22, n. 2, pp. 91–98; 1944.

[2] D. D. Grieg and A. M. Levine, "Pulse-Time-Modulated Multiplex Radio Relay System—Terminal Equipment," *Electrical Communication*, v. 23, pp. 159–178; June, 1946.

[3] F. F. Roberts and J. C. Simmonds, "Multichannel Communication System," *Wireless Engineer*, v. 22, p. 538; November, 1945.

[4] R. E. Lacey, "Two Multichannel Microwave Relay Equipments for the United States Army Communication Network," *Proceedings of the I.R.E.*, v. 35, pp. 65–70; January, 1947.

[5] B. Trevor, O. E. Dow, and W. D. Houghton, "Pulse-Time Division Radio Relay," *RCA Review*, v. 7, pp. 561–575; December, 1946.

[6] H. T. Friis, "Noise Figures of Radio Receivers," *Proceedings of the I.R.E.*, v. 32, pp. 419–422; July, 1944.

[7] C. W. Hansell, "Radio-Relay-Systems Development by the Radio Corporation of America," *Proceedings of the I.R.E.*, v. 33, pp. 156–168; March, 1945.

of a radio-frequency carrier is utilized, noise-reduction properties are the same as at video frequencies. Where frequency modulation of the carrier is by means of time-modulated pulses (frequency-shift keying), the video-frequency relationships with respect to noise are likewise similar, but reduced by the ratio of the improvement factor attributable to frequency-modulation transmission.

A gate limiter will remove noise amplitude modulation as well as noise occurring between pulses. The following discussion assumes an idealized pulse that builds up to maximum amplitude and decays in a time determined by the transmission bandwidth. Under these conditions, both the noise and signal pulses can be represented approximately by triangular shapes.

The time displacement of the pulse edge caused by a noise impulse is shown in Fig. 1 as $a'b'$. A narrow gate limiter is set at the pulse amplitude corresponding to the peak of the noise. Hence, as shown, the signal-to-noise ratio at the threshold level represented by time modulation is improved over that obtained at the receiver input by the factor D/G, where D is the modulation displacement and G is the build-up time. It is well known that the frequency band necessary to support a pulse build-up time G is inversely proportional to G. Therefore, the signal-to-noise improvement ratio is directly proportional to the frequency bandwidth of the receiver, provided the transmitted bandwidth is equal to or greater than the receiver bandwidth.

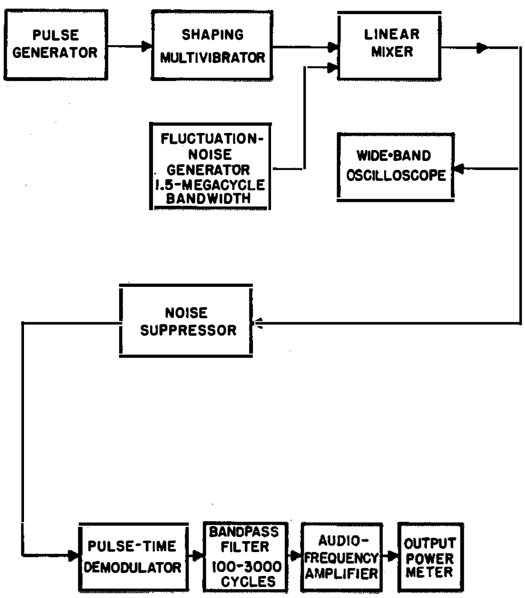

Fig. 2—Block diagram of pulse-time-modulation noise-test setup.

It should be pointed out that it is not possible to derive in a simple manner the exact constants of proportionality. In practice, purely triangular pulses are not common, nor is the pulse edge truly linear. Furthermore, it is necessary to know the relation between the equivalent noise peak (N) and the root-mean-square noise voltage.

It is interesting to note that the input signal-to-noise ratio in a time-modulation system in which the frequency band is optimum for a given pulse width is constant with respect to the frequency band, and depends only on the average power. Corresponding to the increase in noise amplitude with increasing bandwidth, the pulse amplitude will be increased in the same proportion, because the narrower pulse for the same average power will represent greater peak power. Thus, for a given average power, the improvement in signal-to-noise ratio that can be realized with time-modulated pulses is proportional to the frequency band, as is the case with frequency modulation, but, unlike frequency modulation, the improvement ratio continues to increase with increasing bandwidth.

This analysis of pulse-time modulation is based on a demodulation system in which the pulse edge defines the pulse timing. It is possible for the leading and trailing edges of the pulse to be distorted in opposite directions by noise pulses. A further gain of approximately 3 decibels in signal-to-noise ratio may be obtained by utilizing the center of the pulse for demodulation. This gain is realized, however, at the expense of system complication. For example, a system may be visualized whereby both pulse edges are demodulated and the outputs added to reinforce the modulating signal, but partially cancel noise.

Many types of noise pulses, which run the gamut of all shapes and variations in time consistent with the bandwidth of the receiver, might be imagined. So far as their interfering effects are concerned, only those edges of noise pulses that actually coincide in time with the signal pulse edge will cause an audio-frequency noise output.

1. Noise Tests

The types of noise suppressors described in this paper that have been used in pulse-time-modulation receivers are gate limiters, differentiators, and multivibrators. Tests were conducted wherein a train of time-modulated pulses was transmitted to a pulse-time demodulator over a wire link in which the noise-suppression circuit under test was inserted.

A block diagram of the apparatus is shown in Fig. 2. The wide-band fluctuation noise contained frequency components from 30 cycles per second to 1.5 megacycles. The noise generated in a resistor was amplified by an 11-megacycle intermediate-frequency amplifier having a band-pass characteristic of ± 2.5 megacycles. The band of noise at the intermediate frequency was transposed to video frequency and the bandwidth limited to 1.5 megacycles by an adjustable output filter.

In carrying out the noise tests, provision was made for substituting a pulse-interfering source for the fluctuation-noise generator. The pulse-interference source consisted of a combination multivibrator, differentiator, and shaper circuit. This device generated pulses of a constant width and with a repetition rate continuously variable from 250 to 1000 pulses per second. The amplitude of this interfering signal was continuously adjustable without destroying the pulse shape.

The double-gate limiter, shown in Fig. 3, consisted of two pentodes having individually adjustable grid-bias controls to determine the position of the upper and lower levels of limiting.

A resistance-capacitance type of differentiator was used in conjunction with a limiter, so that the leading edge of the signal pulse could be selected and demodulated. The multivibrator was of conventional type, as may be seen from Fig. 4. Input and output coupling stages isolate the multivibrator from external effects. In addition to variable time constants, an input attenuator was provided to control the amplitude of the synchronizing signal supplied to the multivibrator. This input attenuator was constructed so that neither the input pulse shape, output pulse shape, nor the multivibrator time constant was affected during manipulation.

The characteristics of the pulse-time transmission system were as follows:

Pulse repetition rate (pulses per second)	12,000
Pulse period (microseconds)	83
Modulation displacement (microseconds)	± 8
Pulse build-up time (microseconds)	0.75
Pulse decay time (microseconds)	1.5
Pulse width at base (microseconds)	2.5
Audio modulation frequency (cycles per second)	400
Demodulator audio-frequency pass band (cycles per second)	100 to 3000

Oscillographic comparison was made of the input pulse signals and interfering noise. For

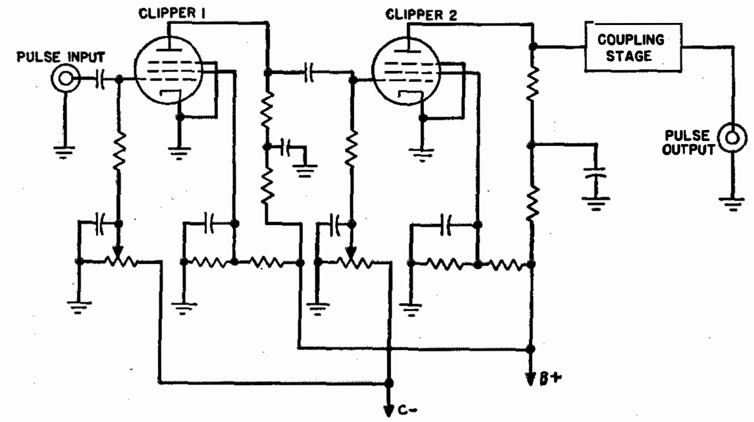

Fig. 3—Double-gate limiter used as protection against interference.

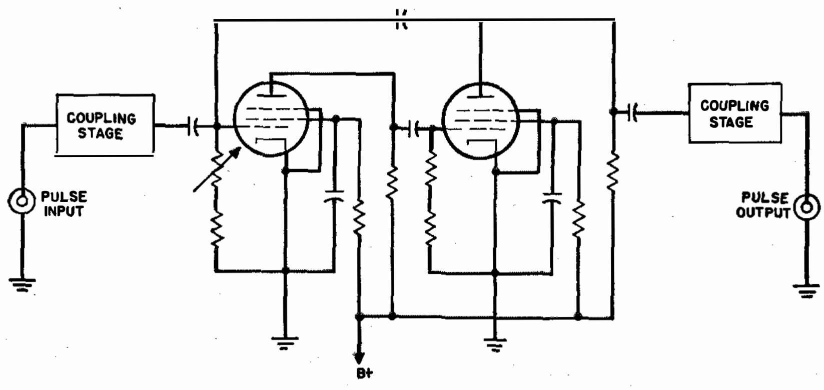

Fig. 4—Multivibrator used as protection against interference.

this measurement, the horizontal sweep voltage usually was removed from the deflecting plates, and the magnitude of the vertical traces compared.

Peak values of noise were measured in this manner for convenience. A comparison measurement by means of a thermocouple was made to determine the ratio of the peak value of noise as measured by the oscilloscope to the root-mean-square value, and this ratio was found to be 3.5.

2. *Fluctuation Noise*

In the first test on fluctuation noise, the output signal-to-noise ratio was compared to that at the demodulator input without noise-suppression devices. The results are shown graphically in curve *A* of Fig. 5. The input signal-to-noise ratio is given in terms of peak amplitude, because of the oscillographic method of measurement. Thus, 6 decibels here corresponds to a noise peak equal to one-half the modulation pulse peak.

The output signal-to-noise ratio is shown to be proportional to the input ratio with no improvement. Under these conditions, the output signal contains noise for two main reasons. First, noise is introduced directly as amplitude modulation in the demodulator. Secondly, some noise is introduced as time modulation because of the inherent nonlinearity of the demodulator. It is obvious that, in a multichannel system, where only selected groups of pulses are applied to the demodulator, some improvement would be obtained because a large portion of the pulse-repetition period would be blanked out. Thus, only noise appearing within the time allotted to one channel would affect the output signal. The system used here may be compared to a multiplexed system by extrapolating the results obtained. The output signal is proportional to the modulation displacement, so that, for a maximum modulation displacement of ±40 microseconds, an output signal-to-noise ratio 5 times greater would be obtained. In the multichannel pulse system, the same signal-to-noise conditions for maximum individual channel displacement would be obtained, since the noise power is less per channel by the ratio of channel time to base pulse period. (This example holds, of course, only for the same pulse peak power for both systems.)

Curve *B* of Fig. 5 illustrates the results of using a double-gate limiter to remove amplitude-modulation noise. It can be seen that no critical

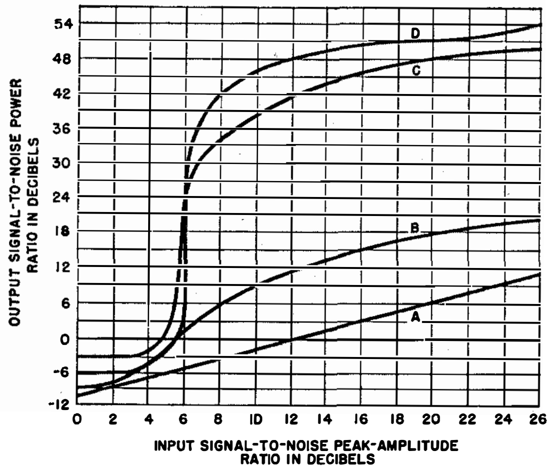

Fig. 5—Output signal-to-noise power ratio plotted against input signal-to-noise peak-amplitude ratio for fluctuation noise and a pass band of 1.5 megacycles. Curve *A* is with no protection. Curve *B* is for a double-gate limiter. Curve *C* is for two double-gate limiters separated by a differentiator. Curve *D* is for three double-gate limiters and two differentiators alternately connected.

threshold occurs, although the output signal-to-noise ratio begins to increase more rapidly when a 2:1 ratio is obtained at the input. There is a gain of about 12 decibels over the ratio obtained in the preceding test. The slight effect of the limiter is accounted for by the presence of width-modulation noise, which may be removed by a differentiator and second double-gate limiter. The action of such devices is shown by curve *C* of Fig. 5. A definite threshold level is obtained, above which noise suppression is considerable. By further differentiation and limiting, the improvement above the threshold is further increased as illustrated by curve *D*.

The function of successive stages of differentiation may be accomplished by a multivibrator that is synchronized by the signal pulses. The multivibrator furnishes a pulse whose leading edge corresponds in time to the leading edge of the synchronizing pulse, and whose trailing edge is a function only of the multivibrator time constants. Only the leading edge is selected for demodulation. In addition, the limiting effect is obtained by the triggering action of the multivibrator. The results obtained with this device are shown in Fig. 6, whence it can be seen that superior improvement is obtained at and above the threshold level.

Further tests were made in which a stage of differentiation was added to the multivibrator. No significant further improvement was noted, showing that the multivibrator entirely removed the width-modulation noise.

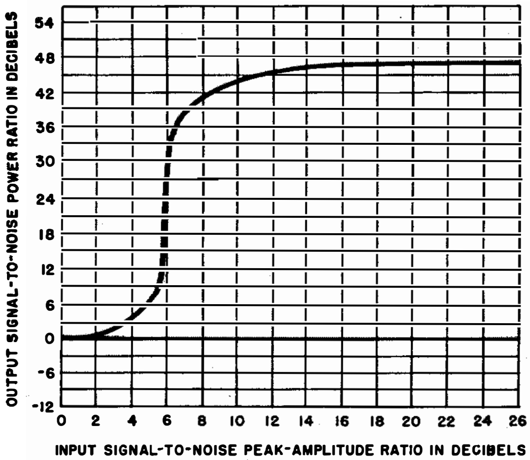

Fig. 6—Conditions similar to Fig. 5, but with a double-gate limiter and multivibrator.

It can be concluded from the foregoing tests that the maximum signal-to-noise ratio improvement can be obtained from a pulse-time-modulation system either by including successive stages of limiting and differentiation, or by incorporating these functions in a multivibrator. In this manner, the reduction of output noise by the elimination of all noise modulations, except that of edge timing, is accomplished.

To determine the noise improvement of pulse-time modulation as a function of the bandwidth utilized, a test was made wherein the video-frequency and noise bandwidths were simultaneously varied, and the output signal-to-noise ratios at the threshold were measured. The resulting curve, Fig. 7, shows that the threshold signal-to-noise ratio is proportional to the bandwidth utilized. From this curve, the empirical constant of proportionality between input and output signal-to-noise ratio may be obtained, since

$$\text{output peak } S/N = \text{input peak } S/N\ (KDF_v),$$

where K is a constant, D is the modulation displacement, and F_v is the video-frequency bandwidth. The value of K can be determined from Fig. 7 to be equal to 7.5.

Since the optimum audio-frequency bandwidth, equal to half the pulse-repetition rate, was not used in these tests, the above equation may be corrected by a factor of $1/\sqrt{2}$. Thus, for the optimum system, we have

$$\text{output peak } S/N = \text{input peak } S/N\ 5.3\ DF_v.$$

Furthermore, the maximum modulation displacement is equal to $1/2f_p$, where f_p is the pulse-repetition rate, and since f_p may equal twice the highest modulation frequency f_a, we may then conclude that

$$\text{output peak } S/N = \text{input peak } S/N\ 1.3\,\frac{F_v}{f_a}.$$

This equation defines the signal-to-noise improvement for the optimum pulse-time-modulation system. The result may be applied to a

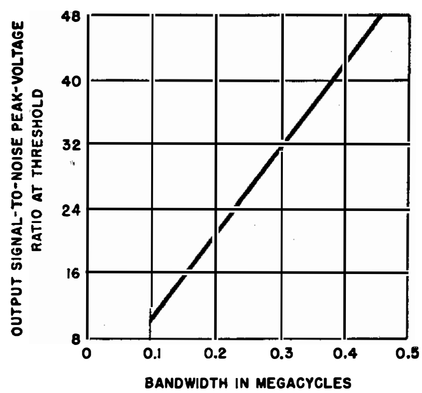

Fig. 7—Signal-to-noise ratio plotted against bandwidth for thermal-agitation noise and signal-pulse displacement of ±8 microseconds.

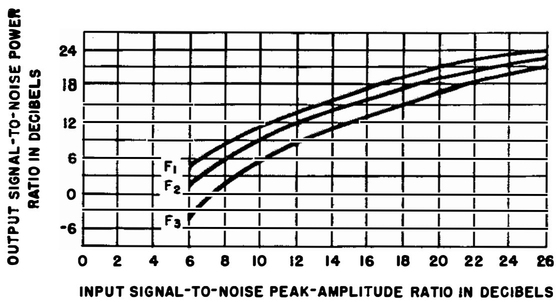

Fig. 8—Output signal-to-noise power ratio plotted against input signal-to-noise peak-amplitude ratio for 3-microsecond noise pulses having repetition frequencies of $F_1 = 250$, $F_2 = 500$, and $F_3 = 1000$. No protection.

multiplexed system by assuming the highest modulation frequency to be the value for one channel multiplied by the number of channels in the system.

3. *Impulse Noise*

Under some conditions, the suppression of impulse noise may be of interest. A study, similar to the foregoing for fluctuation noise, has been made in which the impulse-noise-suppression characteristics of pulse-time modulation have been measured. The noise source for the previous tests was replaced by a generator of pulses whose repetition rate was variable. Fig. 8 illustrates the results obtained without any suppression devices. The output interference varies almost directly with input interference. However, the lower-repetition-frequency pulses cause less interference than those at higher rates. The pulses being of constant width, doubling the pulse frequency increases the noise power by 3 decibels, accounting for the variation shown on the curves. All pulses react on the demodulator, and their fundamental and some higher harmonics are within the pass band of the audio-frequency system. As a result, there is very little, if any, improvement in this unprotected system.

By incorporating a double-gate limiter, a 6-decibel improvement is obtained as shown in Fig. 9. When the signal-to-interfering-pulse ratio is above the 2:1 threshold, only those pulses that

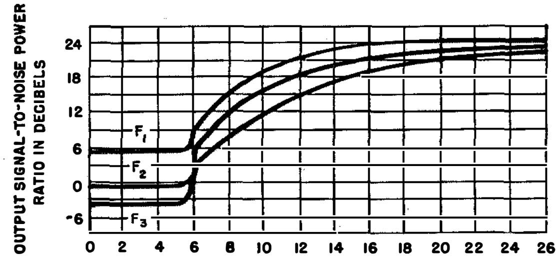

Fig. 9—Conditions similar to Fig. 8, but with double-gate limiter.

occur at the same time as the signal pulses can cause interference. This effect takes place at the beat frequency of the two sets of pulses, causing a distortion of the signal in the same manner as the fluctuation noise, but with a more limited frequency spectrum. Below the threshold, the signal and interfering pulses are limited to the same amplitude, so that the output signal-to-noise ratio is constant.

By adding a differentiating circuit and a second stage of limiting, a sharply defined threshold is obtained. The results of a test using this suppression device are shown in Fig. 10. The steep threshold indicates that the suppression is more complete than that obtained with fluctuation noise.

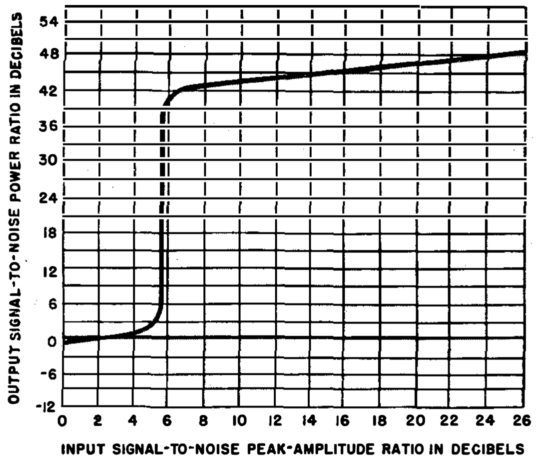

Fig. 10—Conditions similar to Fig. 8, but with two double-gate limiters separated by a differentiator. Noise pulses are at a rate of 500 pulses per second.

4. *Conclusion*

The described tests and results have illustrated the signal-noise capabilities of a pulse-time-modulation system. In addition, the effectiveness of limiters, differentiators, and multivibrators in realizing optimum noise improvement for both thermal and agitation noise and impulse interference has been demonstrated. In addition to normal pulse displacement, noise may result from variations in pulse amplitude, width, and edge slope.

The various devices tested have proved effective in reducing such noise. The uses of these devices are, therefore, indicated to take maximum advantage of the bandwidths utilized for the system. A communication system operating in this manner may then be designed for minimum transmitter power necessary to produce a conservative output signal-to-noise ratio.

Radio Direction-Finding by the Cyclical Differential Measurement of Phase*

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THIS PAPER introduces a new general type of direction-finding and beacon system in which an appreciable reduction of the usual site errors is achieved by the use of aerial structures of wide aperture, the ambiguity normally associated with such systems being resolved by the manner in which the aerials are connected.

Practical forms of the new system generally consist of a circularly disposed array of vertical aerials which are cyclically connected, singly or in groups, by a process of electronic commutation to a receiving device. The basic principle can best be appreciated by considering a single vertical aerial connected to a receiver and caused to move continuously along a circular path in the horizontal plane at a uniform rate. The motion of the aerial would impose a phase modulation on any received signal, and the horizontal direction of arrival of the signal could be determined if this modulation could be related to the law of motion of the aerial.

Several types of direction-finder using the same basic principle are possible; these are outlined and classified. The practical and theoretical advantages of the system are discussed, and two direction-finders, one for use in the very-high-frequency band, the other in the high-frequency band, are described.

The paper is confined to an account of the more important aspects of the subject, attention being paid to the fundamental requirements of the system and the means whereby they are met in practice. The mechanism whereby site errors are suppressed is outlined, and a comparison with the orthodox Adcock types of direction-finder is made, in which it is shown that, just as a phase-modulation communication system has certain inherent superiorities over an amplitude-modulation system, so the method of phase comparison has similar advantages over other forms of direction-finder.

• • •

1. Introduction

Modern ground-station radio direction-finders are generally derivatives of the well-known Adcock system,[1] in the conventional form of which bearings are determined by comparing the amplitudes of the differential output voltages from two opposed pairs of vertical aerials erected at the corners of a square. In such a case the size of the square is limited by considerations of spacing error. This is octantal in form, and if it is not to exceed 2 degrees then the diagonal of the square must be restricted to about $1/3 \cdot 5$ of the shortest wavelength to be served. It is becoming generally understood that site errors can be materially reduced by a substantial increase in the diameter, base-line, or aperture of the aerial system of a direction-finder, and thus the limitation of the Adcock type in this respect is a serious disadvantage.

This paper discusses a new system of direction-finding in which the aperture of the aerial structure is not fundamentally restricted by considerations of spacing error. The system was derived, after suitable modifications during several stages of development, from original suggestions laid down in a joint patent[2] filed by C. E. Strong and one of the authors in 1944. It is interesting to record, however, that as early as February, 1940, H. G. Busignies had described a direction-finder[3] that depended for its operation on somewhat similar principles. The system is considered mainly with reference to direction-finding by reception, but it is equally applicable in principle to radio beacon systems.

* Reprinted with minor additions from *Journal of the Institution of Electrical Engineers*, v. 94, Part IIIA, pp. 705–721; 1947.

[1] Numbered references will be found on p. 71.

2. *Classification of Variations of General System*

The simplest form in which a system involving the cyclical differential measurement of phase can exist consists of a single receiving element moved continuously around some closed path within the radio-frequency field whose direction of propagation it is desired to know. In such an arrangement the path length from transmitter to element varies with time, which implies that the voltage induced in the receiving element will be modulated in phase. The type and extent of this modulation will be associated with the shape and size of the path that the element traverses, and the frequency of repetition of the modulation envelope will be the same as the frequency with which the element completes one circuit of its path. In particular, the modulation envelope will be related in some definite manner in time to the direction of arrival of the received signal. Thus, the bearing of a signal may be elucidated from the time/phase relationship of the phase-modulation envelope. The most obvious time-marking device in the system is the original movement of the element itself. If that motion can be translated into a voltage wave, then the wanted bearing may be derived from the difference between the constant phase of this motion-derived voltage and the bearing-dependent phase of the signal phase-modulation envelope.

For reasons of simplicity it is usual to confine the motion of the element to a circular path, in which case both the motion-derived voltage and the signal phase-modulation envelope become sinusoidal. For other reasons, which will become evident after further discussion, it is often desirable that two moving elements should be used together, and also that the moving elements should be replaced by a series of elements fixed along the same path and switched in such a rhythm that a similar effect is produced.

Discussion of the simpler variations of the general system will be confined to the consideration of the horizontal circular motion of vertical aerials in vertically polarized horizontally propagated radio-frequency fields.

2.1 Cyclic Motion of a Single Element in Free Space

If a single element is continuously and uniformly rotated in free space in a circular path in the horizontal plane, the signal it receives from a distant transmitter will be phase-modulated in a definite sinusoidal manner at the frequency of rotation of the element. If the envelope phase is compared with the (constant) phase of the motion of the element, the phase difference so measured will vary uniformly and unambiguously with bearing angle. The direct measurement of the envelope phase is not a practical possibility, but this difficulty may be overcome by linearly demodulating the phase-modulated signal, thereby producing a sinusoidal voltage having the same frequency and phase as the original phase-modulation envelope. The relative phase angle between this demodulator output and the reference voltage may then be measured in some convenient form of comparator. This angle is equal to the bearing angle of the received signal, or may be made so by an appropriate "zero shift" of the phase of the constantly phased voltage. Thus, if the phase-comparing network actuates some form of phase meter, the meter may be calibrated directly in degrees of bearing. The measurement of the bearing so obtained is inherently accurate because, owing to the linearity of the processes of modulation and of demodulation, no spacing or "repetitive" error can be produced, irrespective of the size of the circle of motion of the element.

2.2 Single-Positional Commutation of Several Aerials

It is not often possible to rotate an aerial round a large circle at sufficient angular velocity to achieve the high rate of phase modulation of the signal that is necessary if the resulting frequency deviation is to be adequate for demodulation purposes. However, if a number of aerials are equally spaced round a circle and connection is made to each one in the correct cyclical time-sequence, the action of rapidly rotating an aerial in the same circle may be simulated. The signal received by a system involving this single-positional commutation of several aerials bears a number of abrupt changes of radio-frequency phase (or "phase steps"), each of which occurs at the instant when the period of activity of one aerial ends and that of the next begins. The magnitude of any phase step depends both on the spacing between the appropriate pair of aerials and on their orientation with respect to the direction of arrival of the signal.

At first sight it appears not only that a very large number of phase steps (and hence of aerials) is necessary if an adequate simulation of a continuously rotating aerial is to be achieved, but also that the existence of any phase step that is greater than $\pm\pi$ radians inherently causes some form of ambiguity of bearing indication. A closer consideration of the problem reveals that, because the phase steps associated with this type of aerial commutation are geometrically related, it is possible to use a reasonably small number of aerials spaced at comparatively large distances without introducing a large repetitive or spacing error into the process of bearing measurement. As will be shown later, in certain applications there may be an advantage in using a spacing greater than one wavelength between adjacent aerials.

The demodulation of the phase modulation imposed on the signal by the aerial commutation involves a slight difficulty. Simple phase-demodulators or discriminators are not linear in action over a very large range of phase change; in fact most of them exhibit a sinusoidal law of instantaneous voltage output against magnitude of the phase step applied. Thus, if the signal reaching such a discriminator bears phase steps in excess of $\pi/3$ radians, the operation of the discriminator becomes noticeably non-linear, and for steps that are greater than $\pi/2$ a smaller output is achieved than for steps of lesser magnitude. The effect of the consequent distortion on the output of the discriminator is to produce a series of harmonics as well as the fundamental-frequency component that alone would be obtained with a linear discriminator. The mathematical treatment of this case is given in Section 12.1(B), and the amplitude relationship between the various harmonics is shown in Fig. 1. In order to secure as much linearity as possible in the process of demodulation, and thereby to restrict the size of the repetitive error, which is the error attributable to the distortion produced by the nonlinear phase discrimination, it is customary to compress, or reduce, the excursion of phase of the signal before applying it to the discriminator. This is achieved by delaying part of the signal for a time equal to, or to a multiple of, the period of activity of one aerial, and then deriving a secondary signal that bears a phase modulation whose extent at any instant is equal to the instantaneous difference between the phase modulations of the delayed and non-delayed parts of the signal. The phase excursion on the secondary

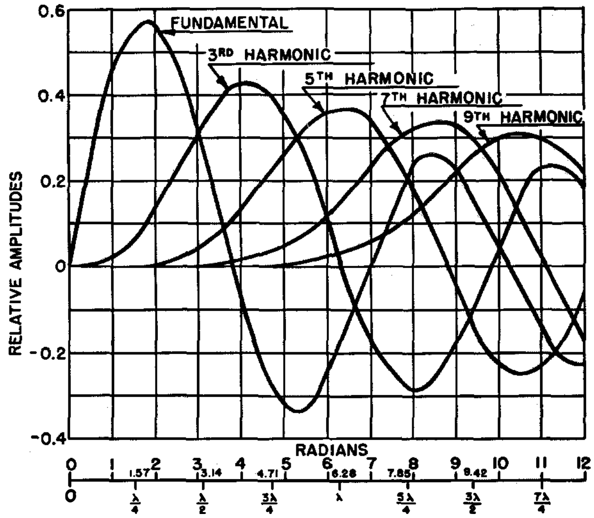


Fig. 1—Harmonic content of audio-frequency output. (A) In pure phase-comparison read abscissae as semi-aperture. (B) In normal (single-delay) phase discrimination read abscissae as spacing between adjacent aerials. (C) In double-delay phase discrimination read abscissae as

$$\frac{(\text{spacing between adjacent aerials})^2}{\text{semi-aperture}}.$$

signal will be smaller than that on the original because a process of differentiation has, in effect, been applied to the original phase modulation. The secondary signal is thus more suitable for demodulation than the original, and less distortion of the output will result. This process could also be adopted where a rotating element is used in conjunction with a sinusoidal phase discriminator (as would be the case in practice). A mathematical analysis of this is given in Section 12.1(C).

Although this method of obtaining the required phase compression by circuit means is adequate for some direction-finding purposes, it suffers from the limitation that if a received signal is not coherent in phase (i.e., the phase of its carrier suffers from random fluctuations owing to the manner of its generation) then the phase of the delayed part of the signal would not be related in any specific manner to that of the non-delayed part. In the equivalent beacon system it may be considered that the practical difficulties involved in obtaining the adequate delay on part of the signal are too great. All such disadvantages are overcome in the extension of the system that will now be described.

2.3 Cyclic Motion of Two Elements in Free Space

In order to effect the necessary compression of the phase modulation without resorting to the intricate and indirect process of obtaining its differential by means of the circuit delay technique, it is desirable to rotate two elements simultaneously in the same circular path at a fixed distance apart. The angle that the chord between them subtends at the centre of the circle should be made equal to the angle through which they would rotate in a time corresponding to the circuit time-delay used in the compression process formerly discussed. Thus, the comparison of the direct output voltages from the two elements will yield the same resultant differential phase modulation as was obtained before, but any random fluctuations of signal carrier phase will be received by both elements simultaneously and will disappear when the differential phase measurement is made.

There is a special case of some importance in which one of the elements is fixed in space instead of being rotated. In this case the rotating element imposes the bearing-dependent phase modulation on the signal, which is then demodulated against the unmodulated signal derived from the fixed element. No phase compression is achieved, but the inability to deal with signals of incoherent phase is removed.

2.4 Double-Positional Commutation of Several Aerials

The practical equivalent of the smooth rotation of two elements involves the commutation of equally spaced pairs of aerials in a circular array. This system of double-positional commutation of several aerials appears to be of real practical value and several important variations of it exist.

First is the group in which the signal from one aerial of an active pair is displaced in frequency by a known amount with respect to the signal from the other. The combination of the two signals in a detecting device yields a secondary signal of constant mean frequency that bears a series of compressed phase steps. The product of the phase demodulation of this secondary signal, after comparison with the phase of a constantly phased wave of the same frequency (which may be derived from the aerial commutation equipment) gives a direct and unambiguous measurement of bearing as before.

Second is the group in which no frequency displacement of one signal with respect to the other is employed. In beacon technique this group subdivides into two sections according to whether a pair of aerials is excited in phase (or anti-phase) or in quadrature. In the latter case no "sense" ambiguity arises. In direction-finders, where "sense" must always be resolved, unless twin reception is used and a frequency shift introduced (thus reverting to classification within the first group), the problem of maintaining the necessary relative amplitudes and quadrature-phasing of the signals from the two active aerials over a wide frequency band presents such difficulty that the system can have little practical value.

A technique has been suggested that appears to offer a solution to this problem, and will apparently overcome several other practical difficulties. Unfortunately, an opportunity to investigate this method experimentally has not yet occurred. A specific example of its application will be described, however, from which the general advantages of any such system will be evident.

Five aerials, *A*, *B*, *C*, *D* and *E*, are equally spaced in order round the circumference of a circle in the usual manner. The normal single-positional commutation order *ABCDE*, *AB*, etc., is replaced by the single-positional sequence *ADBEC*, *AD*, etc., in which the maximum distance (nearly equal to the diameter of the circle) is traversed at each aerial changeover. The intermediate-frequency output of a superheterodyne receiver is then switched alternately into two channels at a rate equal to that of the individual switching of the aerials. Thus, one channel receives output from the aerials in order *A–B–C–D–E–A–B*, etc., whilst the other receives output in order *–D–E–A–B–C–D–*, etc. With a sufficiently narrow bandwidth, each pulse of energy can be caused to increase in duration to twice the "on" period of any one aerial, and thus the time space between the pulses of energy that pass through each channel may be filled. The output trains from the two channels thus become *AABBCCDDEEAABB*, etc., and *CDDEEAABBCCDDE*, etc., respectively, i.e. a double-positional commutation has, in effect, been achieved whose commutation period is twice that actually used.

If a frequency shift has been employed between the two channels, their outputs may be mixed and the compressed phase-modulated output, at the shift frequency, derived and discriminated against the (unmodulated) output of the original shifting oscillator. If no frequency shift has been employed, then the outputs of the two channels may be applied as the push-pull and parallel inputs to a pair of detectors that are connected in opposition to form the final discriminator. The output of such a discriminator consists, as before, of a series of audio-frequency tones whose frequencies are harmonically related to the effective frequency of commutation. Any of these tones may be used to derive the bearing of the signal by comparing its phase with that of the appropriate harmonic of the commutation cycle.

It will be seen that the following advantages exist when such a method of complex commutation is applied. First, the advantages of the use of a twin-channel receiver (or two separate receivers) are gained with the use of a normal single-channel receiver. Second, phase comparison over the full diameter of the aerial system is achieved, thereby increasing the sensitivity of the system to the fullest possible extent. Third, because of the division of the signal into two channels of very simple circuit, after following a common path through the greater part of the receiver, the control of the relative phasing becomes immediately possible and, in fact, quite easy. Thereby one of the most serious difficulties, to which reference has been made, is overcome. Fourth, because an odd number of aerials is used, two commutation cycles are required to effect one complete cycle of aerial combinations, and thus the frequency of commutation is twice that of the fundamental component of the audio-frequency output of the discriminator. Any audio-frequency output due to amplitude modulation imposed on the signal by imperfect aerial switching will have a frequency that is double that of the wanted phase-derived output. Thus, amplitude modulation of this kind is incapable of introducing a distortion of phase of the wanted audio-frequency component, and hence no error of bearing measurement due to this cause can occur. The simpler types of system previously described may, however, be prone to such error, although with good commutation technique the effect is negligible.

In any system of direction-finding dependent on cyclical differential phase measurement, it is found that the repetitive error is due to the distortion of the phase of the wanted component in the audio-frequency output by secondary demodulation products of the same frequency, which exist because of the non-linear characteristic of the discriminator. Such unwanted demodulation products are produced by a complex process of intermodulation, the details of which need not be discussed here (see Section 12.2) except to notice that the interfering output at any frequency is due to the modulation of the inter-aerial switching-frequency "carrier" by the appropriate harmonic in the output. In the system described above, the inter-aerial switching-frequency is ten times that of the fundamental audio-frequency output frequency, and thus if, say, the third harmonic in the audio-frequency output is to be used to measure the bearing, the repetitive error on that measurement will be due to the distortion produced by an intermodulation product at third-harmonic frequency attributable to the action of the seventh harmonic. From the study of a Bessel function chart (Fig. 1) it will be seen that, where the third harmonic is of maximum amplitude, the seventh harmonic is quite small and is thus unable to produce any large repetitive error. In a system consisting of an even number of aerials, say six, and employing the normal commutation sequence, the inter-aerial switching frequency will be only six times the fundamental output frequency, and a correspondingly lower (and stronger) harmonic than before will produce the interfering intermodulation. This will therefore be of greater relative magnitude and a larger repetitive error will occur. The advantage of using an odd number of aerials combined with the more complex commutation order is thereby illustrated.

One of the advantages of the normal double-positional commutation systems is not, however, obtained with this system, because the actual aerial commutation is of the single-positional type and thus bearings of signals of incoherent phase cannot be determined.

Within the scope of the present paper, little more can be said than to acknowledge the existence of systems, similar to those outlined above, in which the bearing information is derived from the phase of the amplitude-modulation envelope rather than from the phase of the phase-modulation envelope.

Apart from systems in which circular aerial arrays are used, it may be mentioned that a direction-finder in which two perpendicular lines of aerials are subject to single-positional commutation at different frequencies has some advantages. In such a system the signals from the two line arrays, by combination after one has been displaced in frequency, produce a secondary signal at the shift frequency; this secondary signal bears a phase-modulation envelope the two components of which correspond to the two original switching frequencies. The relative amplitudes of these two components define the direction of arrival of the signal with respect to the axes of the aerial configuration. The rotational slip of the bearing scale, which is possible when circular aerial arrays are used, cannot occur in this case because the axes of the scale are permanently defined by an absence of modulated output from one line of aerials, owing to the fact that the signal is arriving in line with the other array.

3. *Utilization of Products of Modulation*

As mentioned in Section 2.2, all practical phase discriminators that are suitable for the demodulation of signals bearing large phase steps (i.e. greater than $\pi/3$ radians) tend to exhibit a sinusoidal law between output voltage and phase step applied. It may be shown [Section 12.1(A)] that in such a discriminator the product of demodulation of a signal from a commutated circular aerial array is a series of audio-frequency tones whose frequencies are multiples of the frequency of commutation and whose relative strengths are equal to the Bessel functions of the first kind, of order equal to the harmonic number of the tone, and of argument equal to the greatest phase step imposed on the discriminator, measured in radians.

3.1 Operation on the Fundamental Frequency of Switching

It has been the general practice to select the fundamental frequency component of the discriminator output for use in the determination of the bearing of a signal. Where only one stage of differentiation is imposed on the phase modulation of the signal (i.e. that due to the discriminator) it will be seen [Fig. 1(B)] that the greatest amplitude of output is achieved when the spacing between successively commutated aerials is somewhat greater than $\frac{1}{4}\lambda$, i.e. when the sinusoidal discriminator is dealing with phase steps that are in excess of $\pi/2$ radians. It follows that it is possible to achieve a good output for spacings considerably greater than the optimum value, although ultimately the output falls through zero amplitude to invert in phase.

As mentioned, it is possible, by recourse to a further stage of differentiation of the phase modulation imposed on the signal by the aerial commutation, to produce a further compression of the modulation, thereby involving the discriminator in the demodulation of smaller phase steps than hitherto. By this means, a large aperture for the aerial system and hence large spacing between adjacent aerials become possible because the excessive phase steps produced by the commutation are compressed to an extent that permits unambiguous demodulation by the final discriminator. An example of the practical application of this method will be discussed at a later stage when the equipment is being described.

3.2 Operation on Harmonics of the Frequency of Switching

If we refer to Fig. 1(B), in which the relative amplitudes of the various harmonics of the output of a sinusoidal discriminator are plotted against the maximum phase step imposed by the aerial commutation, we see that the amplitude of the fundamental component of the output rises to a maximum and then falls through zero to increase again in an anti-phase condition according to a Bessel function law. When the aerial separation is so large that the fundamental component is greatly beyond its maximum amplitude, it will be seen that other harmonics of the output are of useful magnitudes. Thus, if one of these harmonics is utilized, instead of the fundamental, to produce the bearing information, it is possible to use an aerial spacing, and hence a system aperture, larger than that which would be the optimum for operation on the fundamental. There is one inherent disadvantage in operating on a frequency other than the fundamental because, whereas for one rotation of bearing the phase of the fundamental component would rotate once, the phase of, say, the third harmonic will rotate three times. Thus, in this particular case a three-fold ambiguity of "sense" appears. This may be resolved by obtaining a coarse but unambiguous indication of bearing, either by utilizing the fundamental component obtained from the discriminator, which will be of small amplitude, or by using a larger output at the fundamental frequency obtained by beating together the appropriate relatively strong harmonics in the discriminator output. On the other hand, the fact that operation on the harmonic frequency causes the bearing indication to rotate a related number of times as fast as normally, leads to the great advantages that higher reading accuracy is possible, and that the increased aerial aperture that is permissible decreases the susceptibility of the system to site errors.

4. *Advantages of Systems of Large Aperture*

The considerable susceptibility of most direction-finders to site errors is a very unfortunate feature. It is possible to reduce such errors by a process of increasing the directivity of the aerial system. By this means, obstacle reflections, unless they are approaching from very nearly the same direction as the signal and thus fall within the directive lobe of the steerable aerial system when orientated for the most favourable reception of the signal, will suffer an attenuation with respect to the signal and will thus be reduced in their capacity to produce an error. In the special condition, when the signal and the necessarily much smaller siting interference are arriving from very nearly the same direction, there can be little error in any case and the need for a further degree of directivity to resolve between them does not occur.

All systems that hitherto have been suggested to achieve high directivity have involved the use of large aerial apertures, although it is theoretically possible, as will be indicated later, to achieve an equivalent degree of error suppression by the use of small apertures if the signal-to-noise ratio is sacrificed by amounts that would be considered intolerable in practice. The use of simple wide-aperture systems necessarily involves a high degree of ambiguity owing to the multi-lobar directive pattern produced. It is, however, possible to resolve such ambiguity by utilizing a more complicated aerial system in which a large number of aerials is used to fill the aperture. It is then found that site-error reduction occurs more rapidly with increase of aperture than in the case of the simple system when the reduction is a linear function of aperture (assuming a reasonable initial value of that dimension).

4.1 "Capture" Effect and Signal-to-Noise Ratio Considerations

The phase modulation imposed on a signal by the rotation of the receiving aerial in a circular path is proportional to the radius of that circle. As the size of the circle is unlimited, the advantages of the corresponding absence of limitation on the extent of the phase modulation are exactly equivalent to those gained by the same phenomenon in the case of phase-modulation communication technique.

In this connection it is interesting to compare the orthodox type of Adcock direction-finding system,[1] where the spacing between the aerials determines the amount of side-signal, with the simple amplitude-modulation type of communication, where depth of modulation of a carrier wave determines the amount of side-signals (or sidebands). In both cases the permissible amount of modulation is restricted by requirements of linearity.

In the new system, a high signal-to-noise ratio, the suppression of interference, and the suppression of the effects caused by the signal when it travels along secondary paths (site errors) are all achieved in the same manner as for similar phenomena in phase-modulation communication technique. For signal phase excursions greater than about one radian, signal-to-noise ratio and the suppression of error are both proportional to phase excursion and hence to aerial aperture. In other words, the "capture" effect, or favourable amplitude discrimination of a signal of large phase excursion, operates in the same way for each system and provides the same improvement.

Smooth rotation of an aerial in a large circle provides a signal of sinusoidal phase excursion modified by rapid perturbations caused by the arrival of secondary signals of small amplitude from directions other than the correct one because of reflection from siting obstacles. If the phase excursion is large, such phase perturbations have little effect upon the phase of the fundamental component of modulation.

In a commutated system, the effective phase excursion of the signal evidently depends, in some measure, on the number of phase steps that occur during one complete cycle of commutation. If, for example, the phase steps never exceed 90 degrees, the perturbations of phase of the radio-frequency field caused by unwanted signals due to reflections from obstacles may be recorded with sufficient frequency for the maximum reduction of error to be obtained. Fewer and larger steps may prevent the cancelling-out of the effect of reflections, and thus may not cause suppression of the error by the maximum amount.

When the aerials are spaced by not more than a quarter wavelength apart, error suppression must be roughly proportional to the size of the aperture of the aerial system. Where the spacing between adjacent aerials is large, error suppression is proportional to the number of aerials used.

As mentioned, large phase steps may be reduced by a process of differentiation during reception. The site-error suppression can then be estimated from the effective aperture of the "compressed" signal, and is thus less than that indicated by the actual dimensions of the aerial system. This lessening of the suppression of the site errors is counterbalanced by the reduction of the repetitive errors consequent on the use of a phase-compressing device.

It is also a fact that in a system that depends on single-positional commutation of the aerials, only signals of coherent phase will cause a bearing to be indicated. Signals of incoherent phase can produce no effect, and thus interference of this nature is suppressed.

4.2 Analysis of Reduction of Site Error

It would seem appropriate to analyse the reduction of site error consequent on the use of the particular types of system now under discussion by considering the case of a smoothly rotating aerial. In any specific commutated system, the necessary information may be deduced from the results of the investigation of the general case.

Let it be assumed that an aerial is rotated over a circle of radius n electrical radians (i.e. $n/2\pi$ wavelengths) in the radio-frequency field to be examined.

The correct or wanted signal yields a carrier wave and various sidebands corresponding to the rotation, their relative amplitudes being exactly in accordance with the corresponding Bessel functions of the first kind, of argument equal to the radius n radians. Whatever the radius, an infinite number of sidebands are produced, although, in general, an increase in the radius tends to increase the amplitude of the higher-order sidebands.

Theoretically, the signal bearing could be determined by comparing the phase of the demodulation product of any pair of sidebands with the phase of the corresponding harmonic of the aerial-rotation frequency. In such a case the first pair of sidebands yields the bearing without ambiguity, and the m-order pair yields the bearing with m positions of ambiguity.

Now consider the effect of a smaller signal, such as would be produced by a single reflecting obstacle, that is arriving from a different direction from that of the "correct" signal. Let it be supposed that the amplitude of this interference is r times that of the "correct" signal (where $r \ll 1$). Owing to the rotation of the aerial, an exactly similar set of frequency components to those produced by the "correct" signal will be produced by this unwanted signal, except that each and every component will have an amplitude only r times that of the corresponding original "correct" one. The effect of the combination of the two waves is to produce summation components, which may be distorted from their correct amplitude and phase by the maximum amounts $\pm 100r$ percent and $\pm \arcsin r$ radians respectively. In estimating the bearing from the phase of the first pair of demodulated sidebands, bearing error can evidently be anything up to

$\pm$arc sin r, which is the figure that corresponds to the normal Adcock system.

Because the magnitude of the sideband phasing error may be as large as $\pm$arc sin r, the error in the estimation of the bearing from the m-order sidebands may be $\pm$(arc sin r)/m. Thus, bearing error can be considered to be independent of aperture, and is suppressed according to the order of the sidebands utilized. A small aperture, however, yields an infinitesimal signal, corresponding to the higher-order sidebands, and thus the signal-to-noise ratio would deteriorate rapidly with increase of the order of the sidebands utilized. Thus, for a practical value of signal-to-noise ratio, the aperture must be increased until sufficient energy is concentrated in the m-order sidebands for the full error suppression of m times to be achieved.

When the aperture is already large, the bandwidth of transmission is substantially proportional to aperture, so that, for constant signal-to-noise ratio, error suppression by phase measurement of a single harmonic is proportional to aperture.

In practice it may be found that the various sidebands are distorted, in phase and amplitude, in different "senses," and by different amounts. If the bearing is measured by means of all sidebands up to order m independently, and if the values obtained are averaged after "weighting" according to the sideband order, then the mean probable error is very much less than $1/m$ times that of the error of the Adcock system. The "averaging" mechanism is exactly that achieved by demodulation of the signal in a frequency discriminator, so that it is probable that the degree of suppression of the mean error is much greater than the proportionate increase of aperture. For any given signal-to-noise ratio, an improvement, which approximates to a square-law effect, may be expected.

The general conclusion is that if an obstacle reflection is present, of amplitude r times the amplitude of the direct-path signal ($r \ll 1$), then the maximum possible bearing error is $\pm$(arc sin r)/m, where m is the highest order of sideband utilized. Good signal-to-noise ratio will be obtained by a phase deviation of about $m+1$ radians (i.e. an aperture of $(m+1)/\pi$ wavelengths).

There are two special cases of relative approach angle between wanted and unwanted signals that are important enough to warrant further consideration.

(A) If the interfering reflection arrives from almost exactly the same direction as the signal, then the maximum error of the normal Adcock system may be less than arc sin r, or even less than (arc sin r)/m, where m corresponds to the largest structure practically feasible. If the normal performance with regard to signal-to-noise ratio is to be maintained, reduction of this error, which in any case is very small, is not possible by means of the new conception. To express the argument in another way, if the interfering reflection arises from approximately the same direction as the signal, the phasing error of the sidebands is proportional to sideband order, so that for normal signal-to-noise ratios all sidebands give the same bearing error. (Note: The large Adcock errors associated with two in-line signals arise only when the signals have almost equal amplitudes—a condition that is outside the scope of this analysis.)

(B) If the interfering reflection differs widely in direction of arrival from that of the signal, the phasing errors of sidebands are random and the maximum error is defined as (arc sin m/r). The use of all the sidebands will tend to suppress the error by much more than $1/m$ times that of the Adcock system, possibly by the square of this function. Thus, error suppression is much greater than aperture expansion.

Returning to the consideration of a commutated circular aerial system, it will be appreciated that, in order to secure a substantial reduction of site errors, it is necessary to utilize the higher-order sidebands and to ensure that those sidebands exist at a reasonable level. This is achieved by increasing the aperture of the system in proportion to the increased order of the sidebands utilized. This increase of the aperture necessitates a corresponding increase in the number of aerials in the system, because the number of aerials determines the reduction in site-error when the inter-aerial spacing is large.

There is some advantage in using a large diameter and few aerials, in that over a very wide band of frequencies the optimum conditions of aperture and spacing between adjacent aerials are thereby caused to occur within the middle portion of the band. Thus, the maximum possible reduction of site error over the whole band is achieved.

5. *Theoretical Design Considerations*

It will be appreciated that there are several special practical considerations that must be taken into account when a direction-finder for a specific purpose has to be designed. In general, when using the process of phase comparison, it is necessary to ensure that the number of aerials, and the spacing between them, are both suitable to their purpose and yield the greatest possible sensitivity without the introduction of a high degree of repetitive error or associated imperfections attributable to aerial interaction.

5.1 Repetitive Errors

It will be realized that, as the aerial commutation is at best a substitute for a continuously rotating aerial, a true identity between the two is not possible. One moves smoothly through the whole field; the other moves intermittently from one position to the next. It would thus be expected that an error should arise from this inadequacy of reproduction of the true effect, since linear demodulation of the phase modulation imposed on a signal is not a practical possibility.

It is, however, possible to calculate the repetitive errors of the system (see Section 12.2). It is found that, for one rotation of bearing, the frequency of repetition of the possibility of maximum error occurs at a rate equal to the number of approach planes of symmetry of the aerial system, i.e. at a rate equal to twice the number of aerials. Although at first sight it would appear that a large repetitive error must occur in any system when the discriminator is subjected to a phase step that lies outside the limits of linearity of its law, this is not the case. When it is taken in conjunction with the related series of steps resulting from the circle commutation, the largest step does not produce an error as large as would be expected. This reduction of error is dependent on the proportionality between the total phase excursion produced by the aerial commutation and the largest single step produced by that commutation. It follows that if the repetitive error is to be small the largest step must be small compared with the total excursion. Thus, the number of steps, and hence of aerials, must be large. If it is required to increase the size of the steps in any system, the number of aerials should be increased also, thus involving a two-fold increase in the system aperture.

The presence of a small interfering signal that is similarly phased for all aerials (e.g. a reflection of the wanted signal from some structure, such as a supporting mast, at the centre of the circle of aerials) will cause an increase of the maximum repetitive error. It is found that an increase in the relative amplitude of this interfering signal causes a disproportionate increase in the error, and the latter soon becomes intolerable.

5.2 Aerial Interaction

By the partial re-radiation of the received energy, each aerial of the system will set up a signal-frequency field of limited strength covering a limited area. Because of the addition of this interacting field to that due to the direct signal, the resultant voltages induced in any aerials of the system lying within the secondary field of another will have phases that will be slightly different from the correct values. Obviously, if the effect is severe the disturbance of phase is sufficient to invalidate the bearing indicated.

Experiment has shown that, if aerial interaction is not to be severe, any aerial, including its earth image, must be less than $\frac{3}{8}\lambda$ long, or, if longer, must be broken into sections less than $\frac{3}{8}\lambda$ in length by the inclusion of series resistors at suitable points. The permissible length varies with the adjacent aerial separation and with the manner in which the aerial is terminated. For instance, an earthed $\frac{1}{4}\lambda$ section will produce a very strong interaction over long distances, whereas with 500 or 1000 ohms inserted in the earthed end the effect is reduced by at least four times, and is noticeable only over distances within a wavelength of the interacting element. If $\frac{1}{4}\lambda$ aerials are used, as is advisable for good sensitivity, and their separation is not large compared with the wavelength, it is imperative to arrange that at the instants in the commutation cycle when an aerial is not being used for reception it shall have as little interactive effect as possible. In general, a high impedance is switched into the base of such an aerial at all times except when it is required to receive the signal.

It is possible to show that interactions that affect the phasing of the various aerials in only a small degree may cause quite intolerable bearing errors.

6. *Very-High-Frequency Direction-Finder*

The original direction-finder, operating on the cyclical differential measurement of phase, was a simple four-aerial equipment intended for use at frequencies in the region of 120 megacycles per second. Single-positional commutation of the aerials was employed, and the results obtained were sufficiently encouraging to warrant the building of a larger system utilizing eight aerials. From the equipment so constructed the present version of the experimental very-high-frequency direction-finder was evolved. Many practical difficulties were encountered on the way, and a great deal of experience was gained of methods by which they could be overcome. Mast interaction, and aerial switching and interaction problems, are obvious examples. The ultimate form of the direction-finder consisted of an 8-aerial balanced-dipole system with a diameter of 3 metres, working over the frequency band 30–125 megacycles, and it would seem appropriate to consider this final equipment in some detail. A block diagram of the system is shown in Fig. 2.

6.1 Aerial Unit

The general appearance of the aerial unit is shown in Fig. 3 and the details of the circuit are given in Fig. 4. It will be seen that a vertical mast, 15 feet high, supported on the roof of a rotatable van, 11 feet high, has at its head a terminating box from which eight horizontal arms radiate. At their outer ends each of the arms supports a vertical dipole whose pick-up is fed, through a balanced 300-ohm feeder contained within the arm, to the central terminating

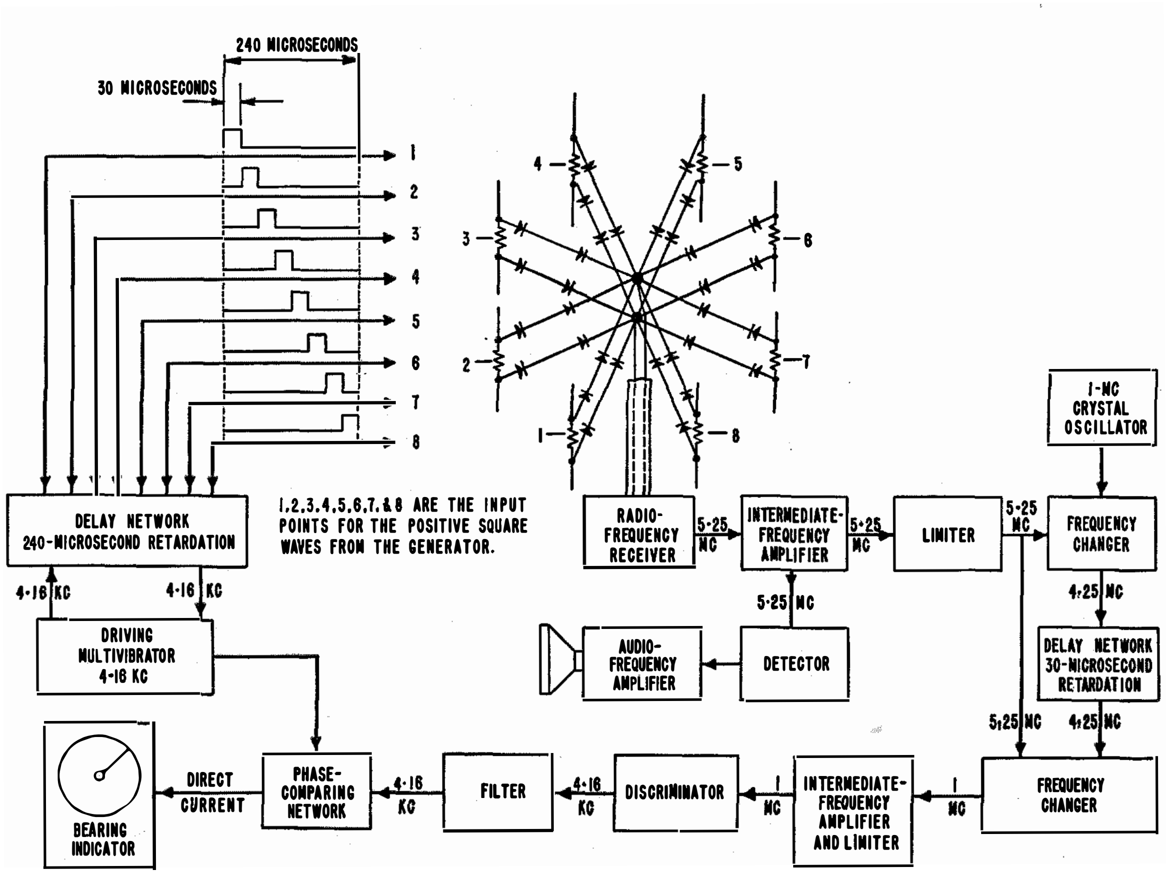


Fig. 2—Block diagram of the very-high-frequency direction-finder.

box in which connection is made to a further feeder, which runs down the inside of the mast and thence to the input terminals of a receiver.

The switching of the aerials is achieved by the use of four small very-high-frequency detecting-type crystals in each arm. One pair, situated at the base of the aerial, serves to disconnect it from the balanced line, and thus to decrease its interactive properties during the periods for which it is not in use; the other pair serves to disconnect the line at its inner end from the common junction point, thereby removing any possibility of interaction effects due to the lengths of the lines. The crystals are pulsed for the appropriate times by means of switching waves generated in the aerial switching unit and passed through screened leads, which run up the inside of the mast and thence to the outer ends of the appropriate arms. Here, connection is made to each pair of crystals through separate series resistors, so that some inequality in the required crystal currents is possible. The crystals are selected for their uniformity, this being important since any differences in the impedances associated with the different aerials and lines will cause different amounts of phase distortion to occur in the radio-frequency trains, thereby affecting the operation of the system. A centre-tapped coil is connected in shunt across the junction of all the lines in the terminating box and is used to provide the necessary earth return for the crystal pulsing currents.

The possibility of using balanced aerials connected to unbalanced lines through balanced-to-unbalanced transformers has been investigated in view of the fact that such an arrangement would halve the number of crystals required, but it was found very difficult to design a transformer to work between the appropriate impedances over

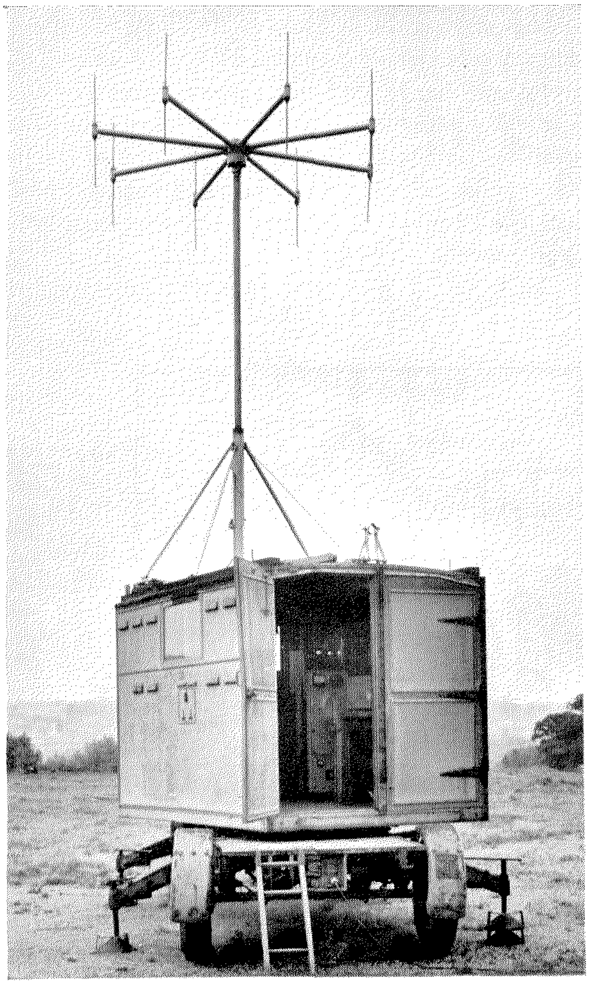


Fig. 3—Very-high-frequency aerial unit.

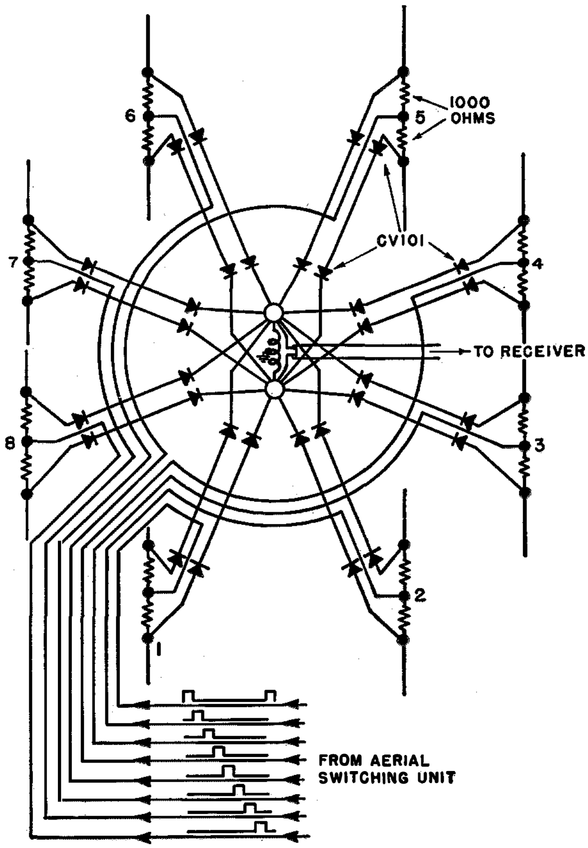


Fig. 4—Circuit diagram of aerial unit.

the wide band required. The other possibility of replacing the two aerial-base crystals by suitable series resistors was also attempted, but it was found that quality of performance was not maintained over the band.

6.2 Aerial Switching Unit

The rectangular positive pulses, of "on/off" ratio 1:7, used to switch the aerial crystals, are obtained in the correct time relationships by means of tappings on a long, high-quality delay network (400 sections, 240 microsecond retardation), which is fed from a 4·16-kilocycle multivibrator of "on/off" ratio 1:7. The frequency of the multivibrator is stabilized by the pulse returned from the farther end of the delay network in the manner that has found application in certain communication devices.[4] The output of the multivibrator is also used as the reference wave in the display-phase comparator (see Section 6.5).

6.3 Radio-Frequency Receiver

The lower end of the aerial feeder is connected to the balanced input circuit of a 28–143-megacycle tunable superheterodyne receiver, and the 5·25-megacycle output from the end of the intermediate-frequency amplifier is passed through a further amplifier to the input circuit of the crystal discriminator unit. A monitoring loudspeaker is also provided.

6.4 Crystal Discriminator Unit

The incoming 5·25-megacycle train from the receiver is "limited" and mixed with a 1-megacycle crystal-controlled oscillation. The 4·25-megacycle beat is passed through a delaying amplifier of 30 microseconds retardation. This delayed train is then mixed with the original undelayed 5·25-megacycle train, and the 1-megacycle beat so obtained, which is of constant carrier frequency, is further limited and then subjected to the action of a sinusoidal discriminator from which an audio-frequency wave of fundamental frequency 4·16 kilocycles is obtained.

This wave is then passed to the display control unit. A side circuit connected to the 4·25-megacycle amplifier is used as a muting control of the gain of a later audio-frequency stage, thereby ensuring that a bearing is only displayed when the receiver is very nearly correctly tuned and that the display is freed from any spurious "off-tune" effects.

As a useful accessory, means are provided

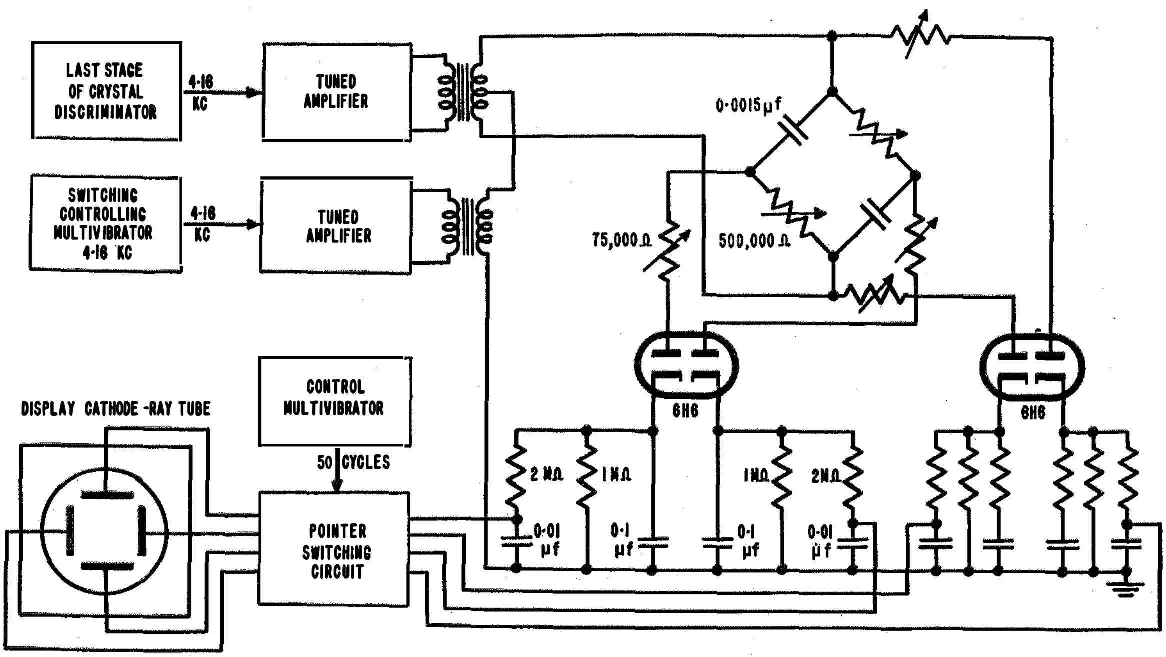


Fig. 5—Essentials of the display mechanism.

whereby amplitude monitoring of the various discriminator processes may be achieved by the use of a cathode-ray tube.

6.5 Display Control Unit

In this unit the 4·16-kilocycle output of the discriminator and the 4·16-kilocycle pulse train from the switching-controlling multivibrator are passed through separate tuned amplifiers and connected respectively in push-pull and parallel at high level to the corners of a phase-splitting bridge (Fig. 5). The four corners of the bridge are connected through series resistors to the anodes of four diodes, and the rectified voltages across the cathode loads of these valves are used to deflect the beam of the display cathode-ray tube. Further shunting diodes, associated with a slow-running multivibrator, serve to produce an intermittent short-circuit of the deflecting voltages so as to cause the spot deflection of the cathode-ray tube to be drawn into a pointer with an origin at the centre of the face of the tube.

6.6 Display Unit

This consists of a commercial type of 5-inch cathode-ray tube with a deflection sensitivity of 40 volts per inch.

6.7 Power Supply Unit

The high-tension direct-current supply, for those parts of the equipment not containing built-in power supply units, provides a current of 300 milliamperes at a voltage of 300. Owing to the steady load, regulation has been found to be unnecessary. The low-voltage supply for heating the filaments of the valves is derived by transformation from the 250-volt alternating-current mains.

6.8 Portable Field Oscillator

A portable field oscillator is used as a signal source for experimental purposes. It will operate with its axis in any position between horizontal and vertical, and covers the full frequency band of the equipment in two ranges, 20–70 and 60–160 megacycles.

6.9 Theory of Discrimination

Consider an aerial system as shown in Fig. 6(a) in which a wave is arriving at an angle of $\theta°$ to the reference line. The amplitude and phase patterns of the "limited" 5·25-megacycle train as it leaves the receiver are shown at Fig. 6(b) and (c). The 4·25-megacycle train after the delay of 30 microseconds will be the same in character as the 5·25-megacycle train, but delayed in time by the period for which one aerial is switched on [Fig. 6(d) and (e)]. The 1-megacycle train, after amplitude limitation, will be also of constant amplitude, but its phase at any instant will be dependent on the phase of the two trains (4·25 and 5·25 megacycles) from which it is derived [Fig. 6(f) and (g)]. In fact, because of the one aerial period (30-microsecond) delay in the 4·25-megacycle path, the phase of the 1-megacycle train at any time will be equal to the change in phase between the radio-frequency input to the receiver at that time and the input when the aerial previous to the one in use was switched on (y_{12}, y_{23}, etc.).

The final discriminator gives an output proportional to the sudden phase change to which it is subjected and lasting for a time dependent on the constants of the circuit. Assuming [Fig. 6(h)] this time to be 30 microseconds, the output of the discriminator will take the form shown.

This stepped audio-frequency output wave is smoothed to a sine wave in the following audio-frequency filter, and its phase is compared with the constant phase of the wave derived from the 4·16-kilocycle multivibrator. A cathode-ray tube is used in conjunction with the phase-measuring device to give an indication of bearing. By taking differing values of θ, it will be seen that the phase of the discriminator output changes linearly with bearing, and is thus an indication of it.

It will be appreciated that the amplitude of the discriminator output at any time, for instance when the changeover from aerial 1 to aerial 2 takes place, is of the form

$$y_{12}-y_{81}=y_2-2y_1+y_8$$

and is thus proportional to the discrepancy between the phase of the output of one aerial (1) and the mean phase of the output of the aerials (8, 2) switched on at each side (in time) of that aerial. Thus a double differentiation has been achieved. In fact, the phase modulation on the signal in the receiver is first differentiated by means of the delay in the $4 \cdot 25$-megacycle path to form frequency modulation, which is then demodulated by the final (frequency) discriminator.

The advantage of using this type of discrimination is two-fold. First, the final discriminator can never be "off-tune," since the 1-megacycle oscillator is crystal-controlled. This implies that there can be none of the multiple tuning effects combined with complete reversals of phase (and hence with complete reversals of indicated bearing) that are associated with the normal type of frequency discriminator. Second, because the process of phase compression decreases the total phase excursion, which in turn implies a reduction of the repetitive error, the diameter of the aerial system may be considerably enlarged before the error is again increased to the maximum tolerable value. The larger diameter of the system thus obtainable results, as has been shown, in a corresponding reduction of the site errors.

As mentioned above, it is possible, without much increase in the di-octantal errors, to extend the phase-changes to which the final discriminator is subjected up to the order of 140 degrees, thereby allowing the further increase of the system diameter to $1 \cdot 25\lambda$, with a corresponding reduction of the site errors.

6.10 Mechanism of Display

The essence of the operation of the display circuits is the action of the differential detector circuits that are associated with the four corners of the phase-comparing bridge (Fig. 5). The "straight through" audio-frequency wave derived from the control multivibrator, being connected in parallel with the bridge, causes each of the four detecting diodes to maintain an equal positive voltage of some considerable magnitude across its load. The "signal" audio-frequency wave, being connected in push-pull to the bridge, will tend, in a similar manner, to set up other direct-current biases across the four diode loads, and it is the combination of these with the constant biases applied by the other channel that produces the differences of potential necessary to deflect the beam of the display cathode-ray tube. In general, if the steady bias of one diode is depressed by the effect of the signal, the bias of the one corresponding to the opposite corner of the bridge will be raised by an equal amount.

It may be shown that if the signal-derived voltage is less than about one-third of the voltage applied through the parallel channel, then the beam of the cathode-ray tube will deflect in a direction almost exactly dependent on the difference of phase of the two voltages, and that one revolution of phase of one voltage with respect to the other will cause the beam of the tube to trace out a curve, which may be made an accurate circle by the careful adjustment of the bridge constants and the series resistances between the corners of the bridge and the anodes of the diodes.

By shunting the deflecting voltages produced in this manner to earth, at 50 cycles, the beam is caused to trace a pointer with its origin at the centre of the tube face. If the pointer is not to be distorted from a straight line by unequal charging effects, care has to be taken that equal time-constants are associated with all the deflecting plates of the cathode-ray tube.

The length, appearance and steadiness of the pointer serve as adequate indications of the strength of the signal and, of course, the orientation of the pointer is a measure of the bearing from which the signal is arriving.

6.11 Performance and Errors

The aerial system has 16 approach planes of symmetry and thus no error is of less order than di-octantal. Such errors as exist are calculable (see Section 12.2), and as the system diameter is larger than that ($\lambda/3 \cdot 5$) of the elevated H-type Adcock derivative, for instance, a marked reduction in site errors is possible, and thus the ability to compensate for a bad site is thereby greatly increased.

The interaction between aerials is not so great as had been feared, chiefly because of the wide separations between them, and is found to become important only when those frequencies are reached at which the length of each half of an aerial is approaching $\frac{3}{8}\lambda$. In that case, the distortion of phase of the voltages from the various aerials caused by the interactive effect may be sufficiently serious to cause a deterioration of the accuracy with which bearings are indicated.

In order to avoid this form of error, the lengths of each of the halves of the aerials have to be kept to $\frac{5}{16}\lambda$ at the highest frequency, which implies that they are only $\frac{1}{16}\lambda$ long at the lowest frequency in the band and are thus very poor collectors of energy, thereby causing failure in the operation of the equipment.

Calculation shows that because, at the high-frequency end of the band, the phase-changes applied to the discriminator are quite large, any signal injected into the aerials by the mast produces a disproportionate repetitive error. Unfortunately, an aperiodic mast interaction was present above 125 megacycles which, although not large, caused sufficient increase in the errors to restrict the upper limit of coverage of the equipment to this frequency. No endeavour was made to reduce the currents in the mast by external means, because whereas the mast interaction appeared to be aperiodic, most counter-measures would be necessarily selective.

The calculation of maximum site error produced by a spurious obstacle reflection of amplitude -20 decibels compared with the true signal was also undertaken, and the results showed such errors to be quite small. For instance, assuming that the first- and third-order sidebands were utilized in deriving the bearing, the maximum site error when the aerial aperture was 1λ was found to be 3 degrees under the most unfavourable conditions of relative approach angle and of relative radio-frequency phase. This is half the value of the site error exhibited by a normal elevated H-type Adcock direction-finder under similar conditions. It should be noted that operation on the third-order sidebands alone yields a maximum error of only 2 degrees. At lower frequencies, where the wavelength dimensions of the aerial system are smaller, the suppression of site error compared with the Adcock type is correspondingly less.

The sensitivity of the system is comparable to and, owing to the large effective aperture, possibly greater than that of the corresponding

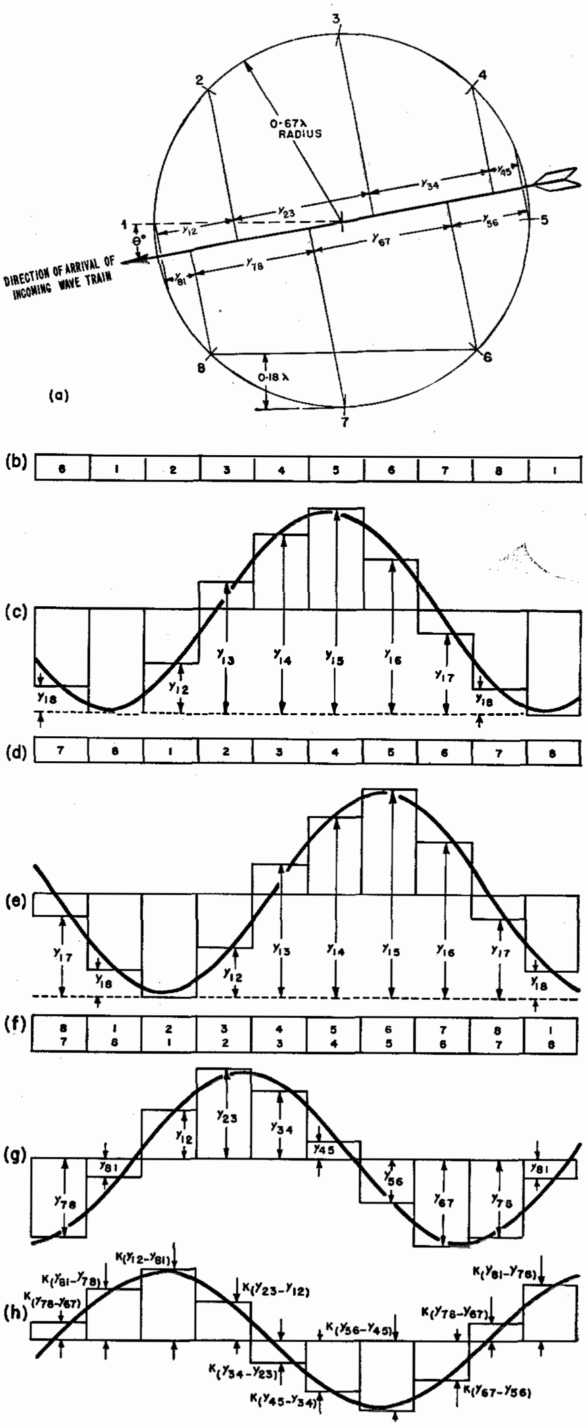


Fig. 6—Operation of the discriminator. (a) Plan of aerial. y_{mn} are phase changes in electrical degrees in proceeding from aerial m to aerial n. (b) Amplitude pattern of 5·25-megacycle train after limitation. (c) Phase variations of 5·25-megacycle train about the mean, showing fundamental component. (d) Amplitude pattern of 4·25-megacycle train after a delay of 30 microseconds has been imposed. (e) Phase variations of 4·25-megacycle train about the mean. Fundamental lags 45 degrees (electrical) behind that of 5·25-megacycle train. (f) Amplitude pattern of 1-megacycle train produced by beating 4·25- and 5·25-megacycle trains. (g) Phase variations of 1-megacycle train about the mean. Fundamental is in quadrature with the mean of the fundamentals of the 4·25- and 5·25-megacycle trains. (h) Amplitude pattern of output from final discriminator, of fundamental frequency 4·16 kilocycles. Phase with respect to a constant-phase wave of the same frequency is $\theta+p$, where p is a constant. Hence, phase of fundamental is a linear function of bearing.

Adcock systems, over that band of frequencies where the dipoles form efficient collectors of energy, i.e. 75–125 megacycles. Below 75 megacycles, the falling efficiency of the aerials, combined with the rather severe switching loss and the decreased effective aperture, causes a decrease in the sensitivity which eventually places a lower limit on the working band at 30 megacycles.

The variation of pointer length that occurs over the band also results from the fact that the wavelength dimensions of the system, and hence the excursion of the phase modulation imposed on a signal, are proportional to the frequency of the signal. With the present arrangements, this variation is small and the pointer length is never less than $\frac{2}{3}$ full scale, for any signal that is strong enough to cause the limiters to work.

The incomplete operation of the limiters in the crystal discriminator unit may allow some of the amplitude modulation due to aerial or mast interaction to be retained as far as the final discriminator, but on demodulation its effect must be to produce an audio-frequency output either in phase or anti-phase with the correct phase-frequency-derived wave, and therefore it cannot (if small in amplitude) affect the phase of the output or hence the indication of bearing.

An investigation of polarization errors showed that the standard wave error (for a signal arriving at 45 degrees to the vertical with a 45-degree transverse tilt in its polarization) was less than 4 degrees over the band of frequencies served.

The ratio of the pick-ups of the aerial system for vertically and horizontally polarized signals arriving horizontally was found to be of the order of 22 decibels over most of the working band. This figure may be less than the actual value, owing to the lack of facilities for the production of accurately horizontally polarized fields.

It was found that only at the highest angles of arrival of the signal did the errors in measurement of the bearing begin to increase beyond tolerable limits. The error due to signals arriving at 60 degrees to the horizontal was of the order of 5 degrees.

A greater frequency band than that finally obtained would be difficult to achieve with the present arrangements, but by using a 12-aerial system of maximum diameter, correspondingly less inherent aerial interaction would permit the use of longer aerials, which in turn would give a better sensitivity at the lower frequencies and thus would enable the band to be extended in both directions.

7. *Proposed High-Frequency Direction-Finder*

The success achieved with the very-high-frequency direction-finder was considered to be sufficient to warrant an investigation into the possibilities of utilizing similar methods in a direction-finder designed to work over the band 2–20 megacycles. It is hoped to proceed at once with the construction of this equipment, and a general description of the design therefore follows. In many ways there will be a similarity to the very-high-frequency experimental model. The block diagram is shown in Fig. 7.

7.1 Aerial System

Twelve vertical aerials are equally spaced around the circumference of a circle 45 metres in diameter. Each aerial consists of a lower single vertical conductor $\frac{1}{4}\lambda$ in length at the highest frequency of operation (20 megacycles), surmounted by a cage $\frac{3}{8}\lambda$ in length. Connection between the two portions is through a resistance of 500 ohms. Each aerial is connected by a buried coaxial feeder to a switching unit housed at the centre of the circle.

The switching of the aerials is achieved by the use of a switched cathode-follower in the base of each aerial feeding the transmission line, and a further switching valve between the inward end of each line and the common junction point. The inner and outer switching valves are pulsed for the appropriate times by switching waves, generated in an aerial switching unit situated in the centre housing and, in the case of the outer valves, passed to them along buried screened cables.

7.2 Aerial Switching Unit

The rectangular positive pulses, of "on/off" ratio 1:11, used to operate the switching valves are obtained in the correct time sequence from a 12-phase phantastron generator, which runs at an overall frequency of 100 cycles. An output from this generator is also used as the reference wave in the display phase comparator.

The reason for the reduction in the switching rate compared with that used in the higher-frequency equipment is that the available bandwidth at these lower frequencies would restrict the possible number of sidebands, resulting from the modulation caused by a higher rate of commutation, to less than the number required for satisfactory operation of the system.

7.3 Radio-Frequency Receiver

The commutated radio-frequency wave derived from the aerials is passed along a buried coaxial transmission line to the operating point, which may be as much as 1000 yards away. There, the received signal is directed into one or more radio-frequency receivers, according to the requirements for multiple-frequency operation.

Each receiver, which is housed in a display console together with its associated bearing-determination and display equipment, is a high-grade tunable communication receiver covering the band 2–20 megacycles. The 580-kilocycle output from the intermediate-frequency amplifier is passed through a further amplifier to the input circuit of the crystal discriminator unit.

Thereafter, the operation of each set of display equipment is similar to that of the very-high-frequency system, the only differences being that the frequencies are correspondingly lower throughout and that finally the bearing is displayed on a larger (12-inch) cathode-ray tube.

7.4 Common Equipment

Testing and monitoring facilities and other circuits (such as the power supply and the termination and dividing circuits at the end of the incoming feeder from the aerials) are housed in a cabinet, which is separate from the display consoles. Facilities may be provided for the repetition at a remote point of the bearing indicated by any of the display consoles in a manner similar to that outlined elsewhere.[5]

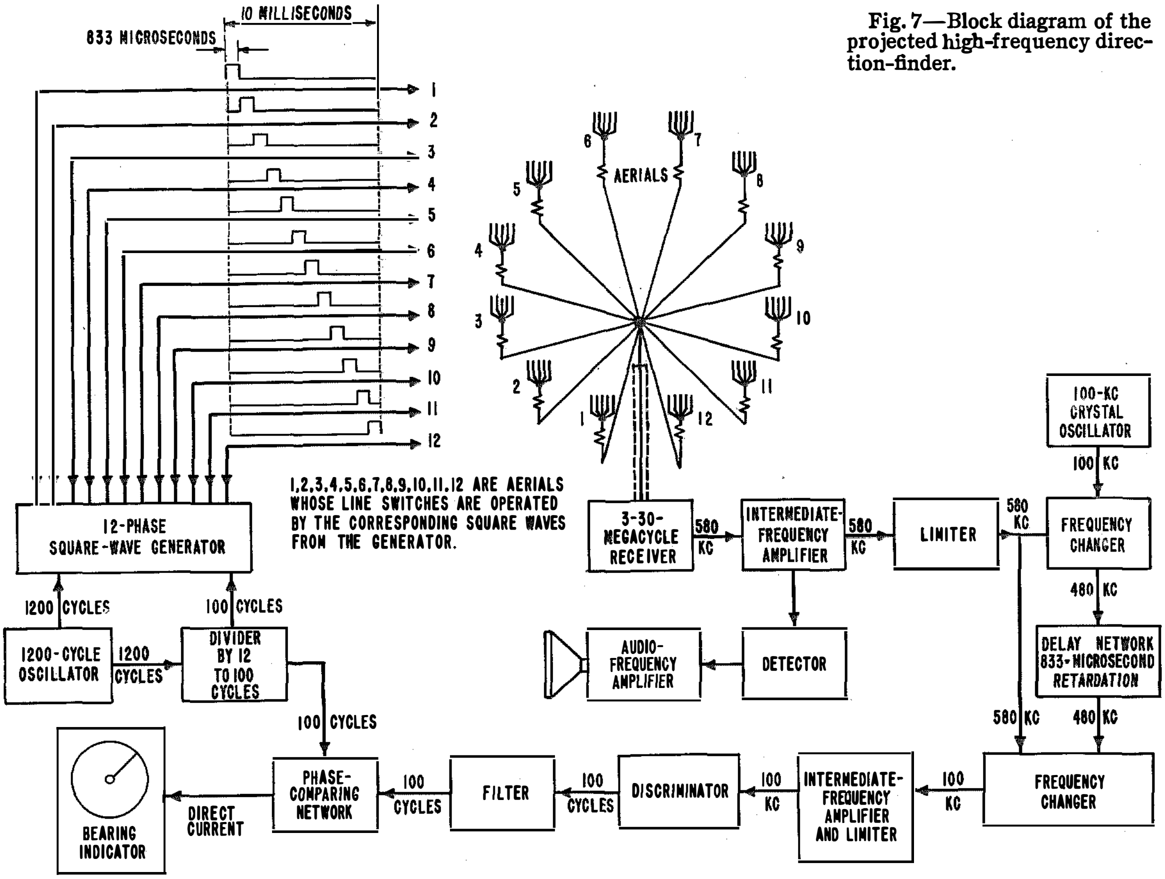


Fig. 7—Block diagram of the projected high-frequency direction-finder.

7.5 Performance and Errors

The aerial system has 24 approach planes of symmetry and thus no error is of less order than di-duodecantal. Such errors as exist are calculable and are small. The aerial system diameter is larger (3λ) at the highest frequency of operation than, for instance, that ($\lambda/3 \cdot 5$) of the elevated H-type of Adcock derivative, and thus an appreciable reduction of site errors should be achieved. In general, errors and other observations on performance are expected to be similar to those of the very-high-frequency equipment.

It is interesting to record that by the use of eleven aerials, instead of twelve, combined with the application of the complex method of commutation outlined in Section 2.4, the error cycle is caused to repeat 44 times per bearing cycle, the maximum error being very small.

8. Beacon Systems

It will be realized that many types of radio direction-finder that come under the general classification of phase-comparison methods may be inverted to become radio beacon systems. In all such systems the obvious necessity arises for transmitting a synchronizing or time-identifying signal with which to compare the phase of the phase-modulation envelope of the signal received from the beacon. In general, it would seem that the easiest method of achieving synchronization would be the transmission of such a signal as an amplitude modulation, which is removed for phase-comparison purposes and which is suitably demodulated in the receiver to yield the reference audio-frequency wave.

In practice, the commutation of large powers at high speeds, necessary to achieve such a beacon, is difficult, especially at the higher frequencies. An experimental beacon has, however, been constructed, which is the counterpart of the very-high-frequency direction-finder described above. By recourse to the measurement of the actual phase of the radio-frequency wave received (rather than of the phase-modulation envelope) it is possible to use much lower switching rates, at which mechanical commutation is quite practicable. An obvious example of such a system is the Post Office Position Indicator.[6]

There also exist types of radio-frequency beacons in which the simultaneous excitation of two transmitting aerials is accompanied by the determination of the phase of the amplitude-modulated pattern produced by the combination of the two particular signals that are being received at any instant. One such system is Navaglobe.[7]

9. Practical Considerations

In many instances it is not possible to obtain theoretical perfection, owing to overriding practical considerations that restrict the full use of all the available facilities. In direction-finding systems operating on the cyclical differential measurement of phase, several such restrictions exist.

The commutation of the aerials produces a train of rectangular pulses of radio frequency that bears a phase modulation, and in order that the full and most accurate output may be derived from the demodulation of the train, it is essential that the shape of the pulses shall be retained throughout the equipment. By defining the "factor of rectangularity" of a pulse as the ratio of total pulse duration to the combined duration of the pulse build-up and decay, it is a useful criterion to require that the factor of rectangularity shall be 5.

The bandwidth of the intermediate-frequency circuits is necessarily controlled by requirements of frequency stability, by the ease with which the receiver is required to tune, and by the practical possibility of producing circuits of such a desired bandwidth. The duration of the pulse is thereby established, since, in order that its shape shall not be distorted, it follows that a pulse must have a duration that is five times the inverse of the bandwidth of the circuits that it traverses. Assuming that one tuned circuit provides a linear time retardation equal to the inverse of four times its bandwidth, it follows that four tuned circuits in series are required to cause a retardation equal to the inverse of their bandwidth. Thus, in such phase-comparison systems as those in which a time-delay equal to a pulse width must be produced, it follows that, for a factor of rectangularity of 5, 20 tuned circuits are required to produce the necessary delay, irrespective of all things other than the value of the factor. If such a number of circuits is considered unpractical, it is necessary to reduce the value of the factor and to accept an inferior result.

It is interesting to note that the commutation-cycle frequency is predetermined, being the inverse of the product of the duration of one pulse and the number of commutator positions, and thus the number of sidebands admitted by a bandwidth five times the inverse of the pulse duration will be five times the number of commutator positions. (The number of pairs of sidebands will be half this quantity.)

The practical difficulties that have to be overcome by the aerial switch have been discussed previously, but it may be of interest to state that a radical improvement in the performance of the very-high-frequency direction-finder could be achieved if a more suitable type of switch than the detecting crystal were available. Not only crystal-resistance variations but also the shunt capacitance affect the conditions of switching by restricting the possible ratio and causing "break-through" of the signal during the inactive periods of the aerials. Variations from crystal to crystal are not easily corrected at all frequencies, and may cause not only an undesirable amplitude-modulation of the signal, but also a considerable degree of misphasing of the aerial pick-ups, with a consequent loss of bearing accuracy. The mechanical and electrical fragility of this type of switch is also a further and not unimportant disadvantage.

10. Conclusion

It is felt that the new field of radio direction-finding that has been opened by the development of the methods outlined in the paper offers many inherent advantages over the classical methods of radio direction-finding and radio-beacon usage. The application of the method to the specific needs of air and marine transport will, it is hoped, bring increased accuracy and reliability in direction determination, which would not otherwise have been obtained.

Finally, the authors desire to thank those colleagues who have helped to provide the subject matter for certain parts of this paper, and who have assisted in its preparation; the Admiralty, and members of the staff of the Admiralty Signal Establishment, who have given considerable encouragement during the work of development; as well as Standard Telephones and Cables, Limited, and in particular Mr. C. E. Strong, Chief Radio Engineer, for granting permission for its publication.

11. References

1. Adcock, F., British Patent 130490; 1919.
2. Earp, C. W. and Strong, C. E., British Patent 594530; 1944.
3. Busignies, H. G., U. S. Patent 2411518; 1943.
4. Levy, M. M., "Some Theoretical and Practical Considerations of Pulse Modulation," *Journal of the Institution of Electrical Engineers*, v. 94, Part IIIA, p. 565; 1947.
5. Cleaver, R. F., "Development of Single-Receiver Automatic Direction-Finder for use in the Frequency Band 100–150 Mc/s," *Journal of the Institution of Electrical Engineers*, v. 94, Part IIIA, n. 15, pp. 783–799; 1947: also *Electrical Communication*, v. 25, pp. 337–362; December, 1948.
6. Mitchell, H. T., and Kilvington, T., "A Time-Multiplex Radio-Frequency Phase-Comparison Method for Navigational Systems."*
7. Busignies, H., Adams, P. R., and Colin, R. I., "Aerial Navigation and Traffic Control with Navaglobe, Navar, Navaglide, and Navascreen," *Electrical Communication*, v. 23, pp. 113–143; March, 1946.

12. Appendices

12.1 Harmonic Content of the Audio-Frequency Output From a Phase Discriminator Having a Sinusoidal Law of Operation When Used to Demodulate the Phase Modulation Imposed on a Signal by a Rotating Receiving Element

Referring to Fig. 8, let the field at D due to the energy radiated by the transmitter be such that the voltage induced in a small receiving element placed there is

$$e_D = E_0 \sin \omega t,$$

where

$\omega = 2\pi f$,
f = frequency of received radiation, in cycles.

Then the voltage induced in a similar element at A, lying on the circumference of a circle ABC of centre D and radius r metres so that AD makes an angle θ with the direction of the received

* Paper to be published in a later issue of Part IIIA of the *Journal of the Institution of Electrical Engineers.*

radiation, will be

$$e_A = E_0 \sin\left(\omega t + \frac{2\pi}{\lambda}.r.\cos\theta\right),$$

where λ = wavelength of received radiation, in metres.

Suppose that the element at A is now caused to rotate at a uniform rate ω_1 in the circle ABC, so that

$$\theta = \omega_1 t,$$

then

$$e_A = E_0 \sin\left(\omega t + \frac{2\pi}{\lambda}.r.\cos\omega_1 t\right).$$

(A) Let it be supposed that the induced voltage in the element is subjected to the action of a discriminator that yields an output at any instant that is equal to the sine function of the phase modulation of the signal. The output will then be

$$D_0 = \sin\left(\frac{2\pi}{\lambda}.r.\cos\omega_1 t\right).$$

Now

$$\frac{2\pi}{\lambda}.r = R,$$

where R is the semi-aperture of the circle of rotation expressed in electrical radians. Therefore

$$\begin{aligned} D_0 &= \sin(R.\sin\omega_1 t) \text{ to a new time origin.} \\ &= 2[J_1(R)\sin\omega_1 t + J_3(R)\sin 3\omega_1 t \\ &\quad + J_5(R)\sin 5\omega_1 t + \ldots \\ &\quad + J_{2n+1}(R)\sin(2n+1)\omega_1 t + \ldots]. \end{aligned}$$

Thus, as the corresponding perfectly commutated case will follow the same law exactly, the product of direct demodulation of a circle-commutated signal is a series of audio-frequency tones, which are of frequencies equal to odd multiples of the frequency of commutation and of relative strengths equal to the Bessel functions of the first kind, of order equal to the harmonic number of the tone, and of argument equal to the semi-aperture of the aerial system expressed in electrical radians. These amplitude relationships are shown in Fig. 1(A).

(B) Let it be supposed that the induced voltage in the element is subjected to the action of a discriminator that yields an instantaneous output equal to the sine function of the difference between the phase modulation of the signal at any time and that of the signal t' seconds earlier. If, say, the element is at A, and t' seconds earlier than it was at B, then the output will be

$$\begin{aligned} D_1 &= \sin\left\{\frac{2\pi}{\lambda}.r[\cos\omega_1 t - \cos(\omega_1 t - \omega_1 t')]\right\} \\ &= -\sin\left[\frac{2\pi}{\lambda}.r.2\sin\frac{\omega_1 t'}{2}\sin\left(\omega_1 t - \frac{\omega_1 t'}{2}\right)\right]. \end{aligned}$$

Now

$$2\sin\frac{\omega_1 t'}{2} = \frac{AB}{r}.$$

Therefore

$$D_1 = -\sin\left[\frac{2\pi}{\lambda}.AB.\sin\left(\omega_1 t - \frac{\omega_1 t'}{2}\right)\right].$$

Now in the corresponding perfectly commutated case, if the associated time delay t' is made equal to the "on" period of any one aerial, then

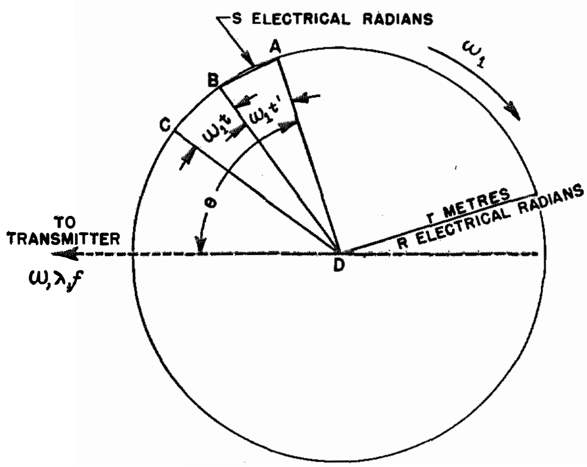


Fig. 8—The ideal rotating receiving element.

the spacing between adjacent aerials will be equal to the chord distance traversed by the rotating element in the same interval of time. Also

$$\frac{2\pi}{\lambda}.AB = S,$$

where S is the spacing between adjacent aerials expressed in electrical radians. Therefore

$$\begin{aligned} D_1 &= \sin(S.\sin\omega_1 t) \text{ to a new time origin.} \\ &= 2[J_1(S)\sin\omega_1 t + J_3(S)\sin 3\omega_1 t \\ &\quad + J_5(S)\sin 5\omega_1 t + \ldots \\ &\quad + J_{2n+1}(S)\sin(2n+1)\omega_1 t + \ldots]. \end{aligned}$$

Thus, in a single stage of circuit differentiation of the signal, the argument of the Bessel functions is equal to the spacing between adjacent aerials expressed in electrical radians [see Fig. 1(B)]. This case corresponds to the use of a normal phase discriminator.

(C) Let it be supposed that the induced voltage in the element is subjected to the action of a discriminator that yields an instantaneous output equal to the sine function of twice the deviation by which the mean of the phase modulation at that instant, and at a time $2t'$ seconds earlier, does not coincide with that at an instant t' seconds earlier. If, say, the element is at A, t' seconds earlier was at B, and $2t'$ seconds earlier was at C, then the output will be

$$D_2=\sin\left\{\frac{2\pi}{\lambda}.r[\cos\omega_1 t-2\cos(\omega_1 t-\omega_1 t')+\cos(\omega_1 t-2\omega_1 t')]\right\}$$

$$=-\sin\left[\frac{2\pi}{\lambda}.r.4.\sin^2\frac{\omega_1 t'}{2}.\cos(\omega_1 t-\omega_1 t')\right].$$

Now in the corresponding perfectly commutated case, if t' is the "on" period for any one aerial

$$2\sin\frac{\omega_1 t'}{2}=\frac{AB}{r}=\frac{S}{R}$$

and

$$\frac{2\pi}{\lambda}.r=R.$$

Therefore

$$D_2=\sin\left(\frac{S^2}{R}.\sin\omega_1 t\right)\text{ to a new time origin.}$$

$$=2\left[J_1\left(\frac{S^2}{R}\right)\sin\omega_1 t+J_3\left(\frac{S^2}{R}\right)\sin 3\omega_1 t+J_5\left(\frac{S^2}{R}\right)\sin 5\omega_1 t+\dots+J_{2n+1}\left(\frac{S^2}{R}\right)\sin(2n+1)\omega_1 t+\dots\right].$$

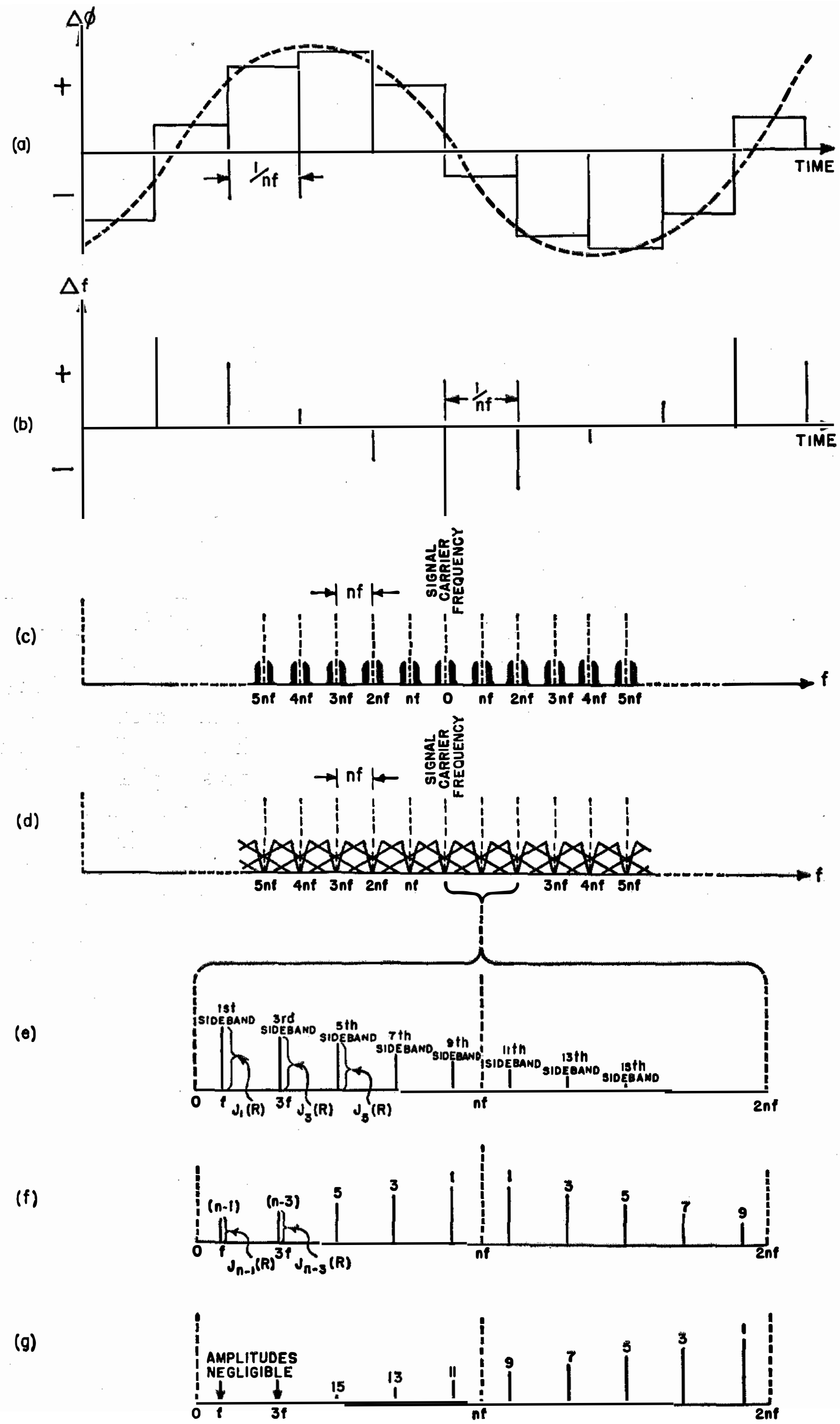
Thus, in the case of a double stage of circuit differentiation of the signal, the argument of the Bessel functions is equal to the ratio: (spacing between adjacent aerials)^2/(semi-aperture). This condition corresponds to that obtaining when one stage of phase differentiation is applied to the signal, as in the case of the very-high-frequency experimental direction-finder, which is described in Section 6 of the paper [Fig. 1(C)].

(D) In the general case of k stages of circuit differentiation, it is found that the abscissae of the Bessel function chart have to be reduced by the factor $(S/R)^k$ from the original values of direct reading in R. The manner of the phase compression associated with differentiation of the modulation is thereby illustrated, since as further stages of differentiation are applied, so the energy of modulation is concentrated to a greater degree in the lower-order sidebands, thus decreasing the relative interactive effects of the higher-order sidebands. The degree of the repetitive error is correspondingly reduced (q.v.) and the system becomes more linear in its action. Eventually, by a sufficient number of successive differentiations, an output will be obtained the energy of which is almost completely concentrated in the first demodulation product, i.e. at the fundamental frequency, and linearity of action must result.

12.2 Calculation of Repetitive Errors

When the smooth sinusoidal phase-modulation imposed on a signal by the smooth rotation of an aerial is replaced by the series of instantaneous phase steps resulting from the corresponding commutated aerial system, the relation between the two will be as shown in Fig. 9(a). These steps of phase may be considered as sharp positive or negative pulses of frequency modulation, as shown in Fig. 9(b). Thus, when an aerial that is rotating at f cycles is replaced by n aerials that are uniformly spaced round a circle of the same diameter and are commutated in turn to give the same frequency of rotation, then the resulting signal modulation is a train of positive and negative pulses of repetition frequency nf. The resulting frequency spectrum of the modulation consists of similar families of sidebands, one corresponding to each of the "carrier frequencies" 0, nf, $2nf$, $3nf$, etc., as shown for small modulations (i.e. for a small-aperture system) in Fig. 9(c), and for larger modulations in Fig. 9 (d), (e), (f), and (g). Owing to the fact that the modulation process is in effect a balanced one, the "carriers" of frequencies 0, nf, $2nf$, $3nf$, etc., are imaginary, having no physical existence, and the lower-order ones are of substantially equal amplitudes. This implies that corresponding (real)

Fig. 9—Calculation of the repetitive errors. (a) Phase-modulation envelopes for the corresponding commutated and rotating aerial systems in a typical case. The dashed line is the envelope due to rotation. (b) Train of pulses of frequency modulation corresponding to the stepped-wave phase modulation of (a). (c) Frequency spectrum of a signal frequency modulated in the manner of (b), showing imaginary carriers and real but discrete families of sidebands (modulation low). (d) As (c), but showing overlapping of families of real sidebands (modulation high). (e) Expansion of (d), showing family of sidebands of imaginary carrier "0" only. (f) Expansion of (d) showing family of sidebands of imaginary carrier "*nf*" only. (g) Expansion of (d) showing family of sidebands of imaginary carrier "2*nf*" only.



sidebands of different (imaginary) "carriers" have the same amplitudes.

Direct demodulation of the sinusoidally swept signal, by a process of beating it with a constantly phased wave of the mean carrier frequency of the signal, yields an output whose frequency components correspond exactly to the sidebands produced by the original modulation, i.e. at frequencies f, $2f$, $3f$, etc. Direct demodulation of the phase-stepped signal in a discriminator, giving a sinusoidal law of output voltage against the degree of phase modulation of the input signal, therefore yields the same frequencies, but each frequency component has several sources, arising, as it does, as the result of the demodulation of the appropriate sidebands associated with the "carriers" whose frequencies are multiples of the pulse-recurrence frequency (nf), i.e. 0, nf, $2nf$, $3nf$, etc.

Owing to this phenomenon, the lower $n-1$, $2n-1$, $3n-1$, etc., sidebands respectively of the "carriers" nf, $2nf$, $3nf$, etc. produce a series of outputs at frequency f by demodulation against the injected mean-frequency wave. In general, the $2n-1$, $3n-1$, and higher-order sidebands are of negligible amplitude and may be neglected. The output at frequency f is thus yielded by demodulation both of the first sideband of the imaginary carrier 0 and of the $n-1$ (lower) sideband of the imaginary carrier nf. As one revolution of bearing rotates the phase of the first sideband of 0 by 2π radians, and the phase of the $n-1$ (lower) sideband of nf by $-(n-1)2\pi$ radians (the sense of rotation of the latter is reversed with respect to the former because one is a lower and the other an upper sideband), it follows that the final output at frequency f is composed of two components whose vectors have a relative rotation of $2\pi n$ radians per bearing cycle. The variation of the phase of the wanted component produced by the presence of the unwanted component is $\pm$arc sin r, where the amplitude of the $(n-1)$ sideband is r times that of the first sideband. (The correspondence of the interfering sideband to a different "carrier" from that of the wanted sideband does not affect the magnitude of the expression, because all the low-order imaginary "carriers" have the same amplitude, and thus their corresponding sidebands are the same size.) It will be seen that the error cycle is $1/n$ of the bearing cycle.

In a similar way, the phase distortion of the audio-frequency output at $3f$, due to the third sideband of 0, results from confusion with the $(n-3)$ lower sideband of nf, etc. In such a case, although the relative amplitude of the unwanted to the wanted component will be larger than when the fundamental frequency f is utilized to display the bearing, and hence the phase distortion will be greater, the fact that the bearing indication rotates three times for one rotation of actual bearing introduces a reduction factor of 3 to the error, which may thus become smaller than the values for operation on the fundamental frequency. It will be appreciated that the maximum repetitive error in any particular case may be calculated from the ratio of the amplitude of the wanted to the unwanted component as given by a chart of Bessel functions, according to which the sideband energy is distributed (see Fig. 1). If the receiving mechanism does not involved direct demodulation, the abscissae of the chart are modified according to the phase compression used, e.g. according to cases (A) and (B) of Fig. 1.

It is interesting to note that the use of an odd number of aerials leads to a positive reduction of the repetitive error. This is because audio-frequency outputs due only to the odd-order sidebands are generated by the process of demodulation. If n is an odd number then $n-1$, $n-3$, etc., are all even numbers and hence no sideband of imaginary carrier nf can produce an interference on the wanted audio frequency of f, or $3f$, etc. The first series of sidebands that can produce error are thus the lower sidebands $2n-1$, $2n-3$, etc., of the imaginary carrier $2nf$. ($2n-1$, $2n-3$, etc., are odd numbers.) It will thus be seen that since the most important interfering sideband for an odd number of aerials is over twice as high (in order) as that for an even number of aerials, the maximum repetitive error is reduced by a very considerable amount.

As an example, let it be assumed that a system of 8 aerials has an aperture of 1.08λ. Then the spacing between adjacent aerials will be $0{\cdot}414\lambda$ and the ratio (spacing of adjacent aerials)^2/(semi-aperture) is λ/π, i.e. 2 electrical radians. Thus where the fundamental output frequency is utilized to derive the bearing and is itself derived by a process involving a double delay, we have:

Magnitude of the fundamental derived directly	0·5767
Magnitude of the fundamental derived by the demodulation of the 7th-order sideband	0·0001749
Ratio of wanted to unwanted component.	3300:1
Arc sin 1/3300 = maximum repetitive error	0·0172 degree

Similarly, for operation on the third-harmonic frequency, the maximum repetitive error is 3·12 degrees of phase, i.e. 1·04 degrees of bearing.

Resonant-Section Band-Pass Filters

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THE SUBJECT of tuned-coupled-circuit filters has been studied and discussed extensively in the literature.[1–4] Complete data are available for the design of two- and three-section filters of this type.[1,3,4] The general case for n sections has been treated by Richards,[2] who shows that an "optimal" characteristic follows a Chebishev* polynomial. In Richards' proof, no special restrictions are placed on the tuned circuits other than that they be lossless and resonant near or at the filter pass-band, or on the interresonator couplings other than that the couplings be nonresonant. The circuits are assumed cascaded, with resonators and couplings alternating in the chain.

In designing and constructing practical filters containing a large number of resonators, it is desirable to know how the various parameters should be adjusted and to what values. It is further desirable that the number of such parameters be held to a minimum.

In the discussion that follows, we shall assume a chain of identical resonators, each coupled by an identical amount to the resonator preceding and the resonator following it in the chain, but not coupled to any other resonator. The nth, or last, resonator is coupled to a resistive load in such a way that the impedance reflected into the last resonator by this coupling is R. Similarly, the first resonator is so coupled to the power source that the generator impedance reflects a resistance R into the first resonator. The common intercouplings between resonators is taken as jX_m (where $j=(-1)^{\frac{1}{2}}$, X_m real) and is assumed constant over the frequency range of interest. The common self-impedance of the lossless resonators is taken as $j2K\alpha$ in the range of interest where k is a real constant, and α is the relative frequency deviation from resonance,

$$\alpha=\frac{\Delta f}{f_0}. \tag{1}$$

For example, if the resonator is a lumped LC series circuit, then $k=\omega_0 L=1/\omega_0 C$. The formula is, of course, only approximate, the approximation depending on the type of resonator; but any simple low-loss resonator fits it with sufficient accuracy.

A schematic of a typical filter is shown in Fig. 1. The determinant of the general network described above, *exclusive* of the reflected generator impedance, is the *continuant.*[5]

[1] C. B. Aiken, "Two-Mesh Tuned Coupled Circuit Filters," *Proceedings of the I.R.E.*, v. 25, pp. 230–272; February, 1937.

[2] P. I. Richards, "Universal Optimum-Response Curves for Arbitrarily Coupled Resonators," *Proceedings of the I.R.E.*, v. 34, pp. 624–629; September, 1946.

[3] K. R. Spangenberg, "The Universal Characteristics of Triple-Resonant-Circuit Band-Pass Filters," *Proceedings of the I.R.E.*, v. 34, pp. 629–634; September, 1946.

[4] M. Dishal, "Exact Design and Analysis of Double- and Triple-Tuned Band-Pass Amplifiers," *Proceedings of the I.R.E.*, v. 35, pp. 606–626, June, 1947, and (discussion) v. 35, pp. 1507–1510; December, 1947: *Electrical Communication*, v. 24, pp. 349–373; September, 1947, and (discussion) v. 25, pp. 100–102; March, 1948.

[5] T. Muir and W. H. Metzler, "A Treatise on the Theory of Determinants," Longmans, Green, and Company, New York, 1933; pp. 516–565.

* This name is spelled variously in English, commonly as "Tchebyscheff".

$$\Delta_n=j^n\begin{vmatrix} 2K\alpha & X_m & 0 & 0 & 0 & \cdots & 0 & 0 \\ X_m & 2K\alpha & X_m & 0 & 0 & \cdots & 0 & 0 \\ 0 & X_m & 2K\alpha & X_m & 0 & \cdots & 0 & 0 \\ 0 & 0 & X_m & 2K\alpha & X_m & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots & 2K\alpha & X_m \\ 0 & 0 & 0 & 0 & 0 & \cdots & X_m & \left(2K\alpha+\frac{R}{j}\right) \end{vmatrix}_n. \tag{2}$$

If $(\Delta_0)_n$ is the continuant defined by

$$(\Delta_0)_n = j^n \begin{vmatrix} 2K\alpha & X_m & 0 & 0 & 0 & \cdots & 0 & 0 \\ X_m & 2K\alpha & X_m & 0 & 0 & \cdots & 0 & 0 \\ 0 & X_m & 2K\alpha & X_m & 0 & \cdots & 0 & 0 \\ 0 & 0 & X_m & 2K\alpha & X_m & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots & 2K\alpha & X_m \\ 0 & 0 & 0 & 0 & 0 & \cdots & X_m & 2K\alpha \end{vmatrix}, \tag{3}$$

then

$$\Delta_n = (\Delta_0)_n + R(\Delta_0)_{n-1}. \tag{4}$$

Writing

$$\rho = \frac{2k\alpha}{X_m} \tag{5}$$

and defining

$$D_n = \begin{vmatrix} \rho & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & \rho & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \rho & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots & \rho & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & \rho \end{vmatrix}_n \tag{6}$$

yields

$$(\Delta_0)_n = (jX_m)^n D_n \tag{7}$$

and

$$\Delta_n = (jX_m)^n \left(D_n - j\frac{R}{X_m} D_{n-1} \right). \tag{8}$$

The input impedance to the network as seen by the reflected generator and its reflected impedance is

$$Z = \frac{\Delta_n}{(\Delta_n)_{11}}, \tag{9}$$

where $(\Delta_n)_{11}$, as usual, is Δ_n less the first column and first row. Clearly,

$$(\Delta_n)_{11} = \Delta_{n-1}$$
$$= (jXm)^{n-1} \left(D_{n-1} - j\frac{R}{X_m} D_{n-2} \right) \tag{10}$$

by (8). Thus (9) becomes

$$Z = jX_m \frac{D_n - j\dfrac{R}{X_m} D_{n-1}}{D_{n-1} - j\dfrac{R}{X_m} D_{n-2}}$$

$$= jmR \frac{D_{n-1} + jmD_n}{D_{n-2} + jmD_{n-1}}, \tag{11}$$

where

$$m = X_m/R. \tag{11A}$$

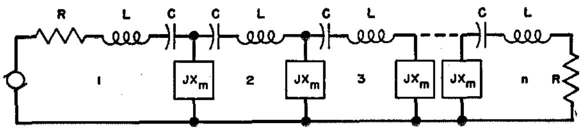

Fig. 1—Schematic arrangement of a typical filter.

It is now desirable to make a further analysis of the continuant (6). First make the substitution

$$\rho = 2 \sin \phi, \tag{12}$$

where ϕ is real for $-2 \leqq \rho \leqq 2$, and complex outside these limits. Then expand D_n in cofactors of the first column, thus:

$$D_n = \rho D_{n-1} - D_{n-2} \tag{13}$$
$$= 2 \sin \phi D_{n-1} - D_{n-2}. \tag{14}$$

(14) constitutes a recursion formula for D_n.

For the first few values of n we have, by direct substitution in (6) and (12),

$$\left. \begin{aligned} D_1 &= 2 \sin \phi, \\ D_2 &= 1 - 2 \cos \phi, \\ D_3 &= 2(\sin \phi - \sin 3\phi). \end{aligned} \right\} \tag{15}$$

Using (14) and (15), we can show readily by induction that

$$D_n = \frac{\sin (n+1)\left(\dfrac{\pi}{2} - \phi\right)}{\cos \phi}. \tag{16}$$

(16) is easily seen to yield (15) for $n = 1, 2, 3$. Assuming that it holds for $(n-1)$ and $(n-2)$, and writing $\theta = (\pi/2) - \phi$, we have

$$D_{n-1} = \frac{\sin n\theta}{\sin \theta},$$

$$D_{n-2} = \frac{\sin (n-1)\theta}{\sin \theta}.$$

Substituting in (14)

$$D_n = \frac{2\cos\theta\sin n\theta - \sin(n-1)\theta}{\sin\theta}$$

$$= \frac{\sin(n+1)\theta}{\sin\theta} = \frac{\sin(n+1)\left(\frac{\pi}{2}-\phi\right)}{\cos\phi}$$

in accordance with (16). Thus if (16) holds for $n-2$ and $n-1$, then it holds for n. Since it holds for $n=1, 2, 3$ by (15), it holds for all positive integers n.

With the help of (16), the input impedance (11) is now expressed as

$$\left.\begin{aligned} Z &= -\frac{mRm\sin(n+1)\theta - j\sin n\theta}{\sin(n-1)\theta + jm\sin n\theta} \\ &= \frac{A+jB}{C+jD} \\ &= R_1 + jX_1, \end{aligned}\right\} \tag{17}$$

where

$$\left.\begin{aligned} \theta &= (\pi/2)-\phi, \\ A &= -m^2R\sin(n+1)\theta, \\ B &= mR\sin n\theta, \\ C &= \sin(n-1)\theta, \\ D &= m\sin n\theta. \end{aligned}\right\} \tag{18}$$

Let η = power insertion ratio of the network
= ratio of power transferred to the load to the maximum transferable power under matched conditions.

It has been shown that

$$\eta = \frac{2R_1R}{(R+R_1)^2+X_1^2} \tag{19A}$$

$$= \frac{4RR_1}{R^2+2RR_1+|Z|^2}, \tag{19B}$$

where

$$|Z| = \text{magnitude of } Z.$$

From (17),

$$R_1 = \frac{AC+BD}{C^2+D^2},$$

$$|Z|^2 = \frac{A^2+B^2}{C^2+D^2}.$$

Substituting these in (19B) yields

$$\eta = \frac{4R(AC+BD)}{(RC+A)^2+(RD+B)^2}. \tag{20}$$

On substitution of (18), this reduces in turn to

$$\eta = \frac{4m^2\sin^2\theta}{2m^2\sin^2\theta + \sin^2(n-1)\theta + 2m^2\sin^2 n\theta + m^4\sin^2(n+1)\theta}. \tag{21}$$

It is more convenient to analyze the reciprocal of this expression defined by

$$\beta = \frac{1}{\eta} = \frac{1}{2} + \frac{1}{4m^2}\left[\frac{\sin(n-1)\theta}{\sin\theta}\right]^2 + \frac{1}{2}\left[\frac{\sin n\theta}{\sin\theta}\right]^2 + \frac{m^2}{4}\left[\frac{\sin(n+1)\theta}{\sin\theta}\right]^2. \tag{22}$$

Equation (22) can be taken as the basic expression for the attenuation characteristic of the system under consideration. Considering the relative exactness of its derivation (as compared with that of classical wave-filter theory, for example), it appears to be a remarkably simple expression. However, while it is true that it affords comparatively simple means for plotting families of curves with m and n as parameters, it does not readily yield useful results obtainable by general analysis. Nevertheless, some conclusions, at least qualitative, may be drawn from inspection of (22).

First, it is to be noted that for those values of ρ for which θ is real the function β is oscillatory in nature, and β cannot be considered to have rejection bands in this range. Thus, this range of values of ρ defines, at least crudely, a "pass band" for the function β. As n increases, the bandwidth for a single oscillation may be expected to become more and more narrow, so that the ambiguity about "cutoff," as related to the points where θ changes from real to imaginary,

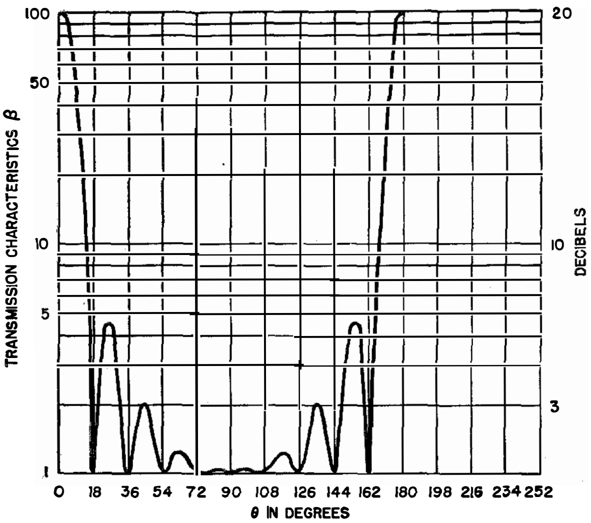

Fig. 2—Transmission characteristics plotted against θ in degrees for a 10-section filter.

becomes less and less. Thus by (1), (5), and (12) we write, tentatively, for large n,

$$\beta W = \frac{2f_0 X_m}{k}, \tag{23}$$

where βW is the bandwidth. If X_m is selected to yield a specified width by (23), then the parameter m is adjusted by varying R (11A). The latter adjustment is obtained by varying the coupling of the first and nth resonators to the power source and load, respectively.

Next, let us try to guess at a value of m to minimize excursions in the pass band. Since θ ranges from 0 to π $\left(\text{because} -\frac{\pi}{2} \leqq \phi \leqq \frac{\pi}{2}\right)$ in the pass band, β is symmetrical with respect to the center of the band $\left(\theta = \frac{\pi}{2}, \phi = 0\right)$. In this region, $\sin \theta \approx 1$ and for small variations in θ decreases only slightly from this value. Therefore, in the neighborhood of $\theta = \pi/2$ we write, approximately, from (22)

$$\begin{aligned}\beta_0 &\approx \frac{1}{2} + \frac{1}{4m^2} \sin^2 (n-1)\theta \\ &\qquad + \frac{1}{2} \sin^2 n\theta + \frac{m^2}{4} \sin^2 (n+1)\theta \\ &\approx \left(\frac{3}{4} + \frac{1}{8m^2} + \frac{m^2}{8}\right) - \frac{1}{8m^2} \cos 2(n-1)\theta \\ &\qquad - \frac{1}{4} \cos 2n\theta - \frac{m^2}{8} \cos 2(n+1)\theta. \end{aligned} \tag{24}$$

For best transmission, β should be as small as possible in the pass band. This suggests minimizing the constant term of (24), i.e., setting $m=1$. Observing the three variable terms in (24), one can note three ranges of possibilities:

A. $m \ll 1$. The significant term is

$$\left[-\frac{1}{8m^2} \cos 2(n-1)\theta\right]$$

and its amplitude is very large

B. $m \gg 1$. The significant term is

$$\left[-\frac{m^2}{8} \cos 2(n+1)\theta\right]$$

and its amplitude is very large

C. $m \approx 1$. Interference patterns are set up between all three variable terms. The *total* amplitude is again a minimum for $m=1$, so that this appears a likely value with which to experiment.

Furthermore, the behavior may be expected to be worse near the edges of the pass band by virtue of the denominator factor $\sin \theta$ in (22); in fact $\sin \theta = 0$ at the edges of the band, but the numerators of the various terms are also zero. Setting $\theta = 0$, we get, easily, at the cutoff points:

$$\beta_c = \frac{1}{2} + \frac{1}{4m^2}(n-1)^2 + \frac{1}{2}n^2 + \frac{m^2}{4}(n+1)^2, \tag{25}$$

and this is seen to be minimal for

$$m = \left(\frac{n-1}{n+1}\right)^{\frac{1}{2}}. \tag{26}$$

This is certainly of the order of 1 for large n; for example, for $n=10$, $m=0.904$.

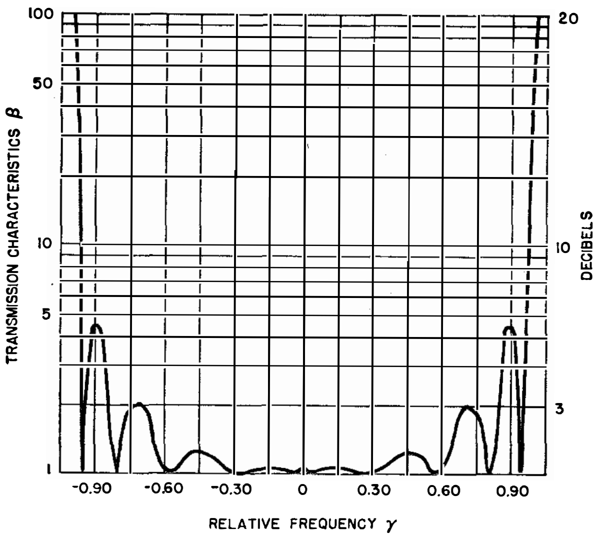

Fig. 3—Transmission characteristics plotted against relative frequency γ for a 10-section filter.

Substituting (26) in (22), we obtain as a fair guess for an optimal characteristic

$$\begin{aligned}\beta(\theta; n) = \frac{1}{2} + \frac{1}{4}\left(\frac{n+1}{n-1}\right)\left[\frac{\sin (n-1)\theta}{\sin \theta}\right]^2 + \frac{1}{2}\left[\frac{\sin n\theta}{\sin \theta}\right]^2 \\ + \frac{1}{4}\left(\frac{n-1}{n+1}\right)\left[\frac{\sin (n+1)\theta}{\sin \theta}\right]^2. \end{aligned} \tag{27}$$

Fig. 2 shows a plot of (27) for $n=10$, and Fig. 3 shows the same function replotted with the abscissa transformed to the coordinate γ, linear with respect to relative frequency and given by

$$\gamma = \frac{k\alpha}{X_m} = \frac{\rho}{2} = \cos \theta. \tag{28}$$

We see that the pass band can certainly not be considered as ranging over all real values of θ; the attenuation is very large at $\theta = 0, \pi$. However, for small deviations from these values, the recovery of the filter is rapid, and the filter is passing practically freely at $\theta=18$ degrees, 162 degrees, $(\gamma=\pm 0.95)$. Thus, except for the smaller dips at $\theta=27$, 45, 153, and 135 degrees, the expected value of pass band is correct within 5 percent. The dips at $\theta=27$ and 153 degrees are about 7 decibels and are considered excessive. The dips at $\theta=45$ and 135 degrees are about 3 decibels and are considered reasonable. The filter is flat within 3 decibels in the range $-0.85<\gamma<0.85$, 15 percent less than hoped for. Inside the peaks at $\theta=45$ and 153 degrees, the filter is extremely flat.

An attempt will now be made to analyze the function (27) to some extent, and to check the general results obtained against the curve of Fig. 2.

The function may be considered as having two sets of regions of major interest; (*a*) the edges of the pass band and (*b*) the region near the center of the pass band.

1. *Edges of Pass Band*

It is sufficient to study the case where θ tends to zero. First, by substitution in (27) at $\theta=0$,

$$\beta(0;n)\rightarrow n^2, \tag{29}$$

which checks with Fig. 2.

Next, from Fig. 2 we guess that the worst excursions of β occur near $\theta=0$. It is desirable to evaluate these excursions in general terms. But first it is useful to note the nature of the function

$$y=\frac{\sin sx}{\sin x},\ s \text{ positive integer.} \tag{30}$$

This function is plotted in Fig. 4 for positive x. It has a maximum at $x=0$ and a zero at $x=\pi/s$. Between these values, it decreases monotonically, The function y^2 has, therefore, a maximum at $x=0$ and a minimum at $x=\pi/s$, with no critical points in between.

Considering now the functions,

$$\left[\frac{\sin (n-1)\theta}{\sin\theta}\right]^2,\quad \left[\frac{\sin n\theta}{\sin\theta}\right]^2,\quad \left[\frac{\sin (n+1)\theta}{\sin\theta}\right]^2,$$

any linear sum of these with positive coefficients has a maximum at $\theta=0$. Certainly up to $\theta=\pi/(n+1)$, the linear sum also decreases monotonically so that it possesses no critical points in this range. This fact is important for the subsequent discussion.

Beyond the point $\theta=\pi/(n+1)$, maxima and minima of β will be governed, for large n, essentially by the numerators of the squared functions, which are rapidly varying functions of θ, and only to a slight extent by the denominators, which are slowly varying.

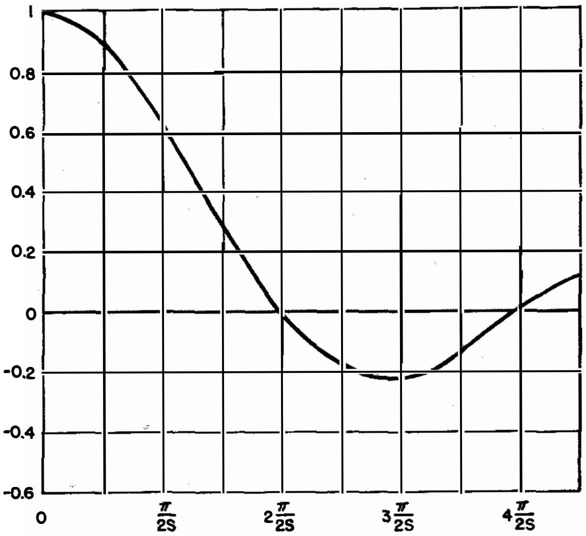

Fig. 4—Plot of function $y=\dfrac{\sin sx}{\sin x}$.

Thus, to locate the maxima and minima of β, we shall rather obtain approximate solutions by considering first the simpler function

$$\beta'=\frac{n+1}{n-1}[\sin (n-1)\theta]^2+2[\sin n\theta]^2 +\frac{n-1}{n+1}[\sin (n+1)\theta]^2. \tag{31}$$

Then, by using the trigonometric identity

$$\sin^2 sx=\tfrac{1}{2}-\tfrac{1}{2}\cos 2sx \tag{32}$$

and dropping constant terms as being of no interest, we obtain the still simpler function

$$\beta''=\frac{n+1}{n-1}\cos 2(n-1)\theta+2\cos 2n\theta +\frac{n-1}{n+1}\cos 2(n+1)\theta. \tag{33}$$

Taking the derivative of β'', equating to zero, and rearranging, results in

$$A\sin (2n\theta-\psi)=0, \tag{34}$$

where

$$A=[n^2(1+\cos 2\theta)^2+(\sin 2\theta)^2] \qquad (34A)$$

$$\psi=\tan^{-1}\left[\frac{\sin 2\theta}{n(1+\cos 2\theta)}\right]. \qquad (34B)$$

Thus, critical points of interest are located approximately at

$$2n\theta-\tan^{-1}\left[\frac{\sin 2\theta}{n(1+\cos 2\theta)}\right]=k\pi, \quad k=0,1,2,\ldots \qquad (35)$$

Near $\theta=0$, the quantity in brackets tends to θ/n; since, for small x

$$\tan^{-1} x\rightarrow x,$$

(35) may be written, approximately, in the neighborhood of $\theta=0$,

$$2n\theta-\frac{\theta}{n}\approx k\pi$$

or

$$\theta\approx\frac{k\pi}{2n-\frac{1}{n}}\approx\frac{k\pi}{2n}\left(1+\frac{1}{2n^2}\right),$$

$$n \text{ large}, \quad k=0, 1, 2, 3,\ldots \qquad (36)$$

Now, as previously pointed out, there can be no critical points of β for $0<\theta<\pi/2(n+1)$. For $k=0, 1$, and for $n\gtrless 2$, the value of θ given by (36) lies in this range. Thus $k=0,1$ are excluded from (36), and the revised set of values reads

$$\theta\approx\frac{k\pi}{2n}\left(1+\frac{1}{2n^2}\right), \quad n \text{ large}, k=2, 3, 4,\ldots \qquad (37)$$

Actually we shall neglect even the second term of this approximation and use

$$\theta\approx\frac{k\pi}{2n}, \; n \text{ large}, k=2, 3, 4\ldots \qquad (37A)$$

For $n=10$, we have $\pi/2n=9$ degrees, so that maxima and minima alternate at 9-degree intervals beginning with $\theta=18$ degrees $(k=2)$ (compare Fig. 2).

For these values of θ, and for k/n sufficiently small, we now have the following series of approximations:

$$\sin\theta\approx\frac{k\pi}{2n}$$

$$\sin(n-1)\theta\approx\begin{cases}\cos\frac{k\pi}{2n}, & k=5, 9,\ldots\\ -\cos\frac{k\pi}{2n}, & k=3, 7, 11,\ldots\\ \sin\frac{k\pi}{2n}, & k=2, 6, 10,\ldots\\ -\sin\frac{k\pi}{2n}, & k=4, 8,\ldots\end{cases}$$

$$\sin n\theta\approx\begin{cases}\cos\frac{k\pi}{4n^2}, & k=5, 9,\ldots\\ -\cos\frac{k\pi}{4n^2}, & k=3, 7, 11,\ldots\\ -\sin\frac{k\pi}{4n^2}, & k=2, 6, 10,\ldots\\ \sin\frac{k\pi}{4n^2}, & k=4, 8,\ldots\end{cases}$$

$$\sin(n+1)\theta\approx\begin{cases}\cos\frac{k\pi}{2n}, & k=5, 9,\ldots\\ -\cos\frac{k\pi}{2n}, & k=3, 7, 11,\ldots\\ -\sin\frac{k\pi}{2n}, & k=2, 6, 10,\ldots\\ \sin\frac{k\pi}{2n}, & k=4, 8,\ldots\end{cases}$$

Substitution of these results in (27) yields as follows.

A. For $k=3, 5, 7, 9,\ldots$

$$\beta\left(\frac{k\pi}{2n}; n\right)=\frac{1}{2}+\frac{1}{4}\left(\frac{n+1}{n-1}\right)\left[\frac{\cos\frac{k\pi}{2n}}{\frac{k\pi}{2n}}\right]^2$$

$$+\frac{1}{2}\left[\frac{\cos\frac{k\pi}{4n^2}}{\frac{k\pi}{2n}}\right]^2+\frac{1}{4}\left(\frac{n-1}{n+1}\right)\left[\frac{\cos\frac{k\pi}{2n}}{\frac{k\pi}{2n}}\right]^2. \qquad (38A)$$

B. For $k=2, 4, 6, 8,\ldots$

$$\beta\left(\frac{k\pi}{2n}; n\right)=\frac{1}{2}+\frac{1}{4}\left(\frac{n+1}{n-1}\right)\left[\frac{\sin\frac{k\pi}{2n}}{\frac{k\pi}{2n}}\right]^2$$

$$+\frac{1}{2}\left[\frac{\sin\frac{k\pi}{4n^2}}{\frac{k\pi}{2n}}\right]^2+\frac{1}{4}\left(\frac{n-1}{n+1}\right)\left[\frac{\sin\frac{k\pi}{2n}}{\frac{k\pi}{2n}}\right]^2. \qquad (38B)$$

Since for small $\frac{k}{n}$, $\dfrac{\cos\frac{k\pi}{2n}}{\frac{k\pi}{2n}}$ is of the order of $2n/k\pi$,

while $\dfrac{\sin\frac{k\pi}{2n}}{\frac{k\pi}{2n}}$ is of the order of unity, it is clear that (38A) corresponds to maximum points while (38B) corresponds to minima of the function. Taking

$$\cos\frac{k\pi}{4n^2}\approx 1,\quad \sin\frac{k\pi}{4n^2}\approx\frac{k\pi}{4n^2}$$

for the range of interest, further simplifies (38A) and (38B) to the forms

$$\beta\left(\frac{k\pi}{2n};n\right)=\frac{1}{2}\left\{\left[1+\left(\frac{2n}{k\pi}\right)\right]+\left[\frac{n^2+1}{n^2-1}\right]\left[\frac{\cos\frac{k\pi}{2n}}{\frac{k\pi}{2n}}\right]^2\right\},\quad k=3,5,7,\ldots \tag{39A}$$

$$\beta\left(\frac{k\pi}{2n};n\right)=\frac{1}{2}\left\{\left[1+\frac{1}{2n}\right]+\left[\frac{n^2+1}{n^2-1}\right]\left[\frac{\sin\frac{k\pi}{2n}}{\frac{k\pi}{2n}}\right]^2\right\},\quad k=2,4,6,\ldots \tag{39B}$$

For $n=10$, we have from (39A) and (39B)
$k=2$, $\beta=1.03$, or 0.1 decibel
$k=3$, $\beta=4.5$, or 6.5 decibels
$k=4$, $\beta=0.98$ (which is impossible since $\beta=1$)
$k=5$, $\beta=1.72$, or 2.8 decibels.

These results are in fair agreement with the data plotted in Fig. 2. They show roughly that the function possesses periodic maxima spaced approximately at equal intervals and given by (39A); it has periodic minima spaced approximately at equal intervals, at which the filter appears to be essentially freely passing. The pass band does not extend quite to the interval $0\leqq\theta\leqq\pi$, but is shortened to an extent depending on the desired flatness. For instance, for $n=10$, if a variation of 3 decibels is permitted in the pass band, then the band limits are almost out to the largest maximum; in terms of relative frequency scale γ, 85 percent of the nominal band may be used.

2. *Center of Pass Band*

Let $\theta=(\pi/2)+\zeta$, where ζ will be taken as being small. Then (27) becomes

$$\beta\left(\frac{\pi}{2}+\zeta;n\right)=\frac{1}{2}+\frac{1}{4}\left(\frac{n+1}{n-1}\right)\left[\frac{\begin{Bmatrix}\sin\,(n-1)\zeta\\ \cos\,(n-1)\zeta\end{Bmatrix}}{\cos\zeta}\right]^2+\frac{1}{2}\left[\frac{\begin{Bmatrix}\cos n\zeta\\ \sin n\zeta\end{Bmatrix}}{\cos\zeta}\right]^2+\frac{1}{4}\left(\frac{n-1}{n+1}\right)\left[\frac{\begin{Bmatrix}\sin\,(n+1)\zeta\\ \cos\,(n-1)\zeta\end{Bmatrix}}{\cos\zeta}\right]^2. \tag{40}$$

In the terms enclosed in braces, the upper quantities are to be used for n odd, and the lower for n even.

Let the range of ζ be restricted so that $n\zeta$ is of the order of unity; say $-2\leqq n\zeta\leqq 2$.

Therefore, ζ is of the order of $1/n$, at most, so that for large n, $\cos\zeta\approx 1$. Then for n odd, (40) is approximately

$$\beta\left(\frac{\pi}{2}+\zeta;n\right)=\frac{1}{2}+\frac{1}{4}\left(\frac{n+1}{n-1}\right)[\sin\,(n-1)\zeta]^2+\frac{1}{2}[\cos n\zeta]^2+\frac{1}{4}\left(\frac{n-1}{n+1}\right)[\sin\,(n+1)\zeta]^2. \tag{41}$$

(Even values of n will yield approximately the same maxima and minima displaced in θ.)

With the help of (32) and some manipulation, (41) can be thrown in the form

$$\beta\left(\frac{\pi}{2}+\zeta;n\right)\approx\frac{1}{2}\left[\frac{2n^2-1}{n^2-1}\right]-\frac{1}{4(n^2-1)}\times\{[-(n^2-1)+(n^2+1)\cos 2\zeta]^2+4n^2\sin^2 2\zeta\}^{\frac{1}{2}}\cos\,(2n\zeta-\mu), \tag{42}$$

where

$$\tan\mu=\frac{2n\sin 2\zeta}{-(n^2-1)+(n^2+1)\cos 2\zeta} \tag{43}$$

and μ is seen to be a slowly varying function of ζ in the range of interest. Equation (42) is seen to consist of a constant term and a rapidly oscillatory one, the coefficient of which is a slowly varying function of ζ. The oscillatory term yields the excursions of β. Making use of the first terms in the power series for $\sin 2\zeta$ and

$\cos 2\zeta$, (42) can be reduced to the form

$$\beta\left(\frac{\pi}{2}+\zeta;n\right)\approx\frac{1}{2}\left(\frac{2n^2-1}{n^2-1}\right)-\frac{1}{2(n^2-1)}(1+n^2\zeta^2)\cos(n\zeta-\mu). \quad (44)$$

In (44), a slight correction has been applied to compensate for the denominator factor $\cos\zeta$ which was removed earlier in the discussion.

The first term in (44) is of the order of unity. The second (oscillatory) term is of the order of $\frac{1}{2}\left(\frac{1}{n^2}+\zeta^2\right)$. For $n\zeta$ of the order of unity, and for large n, this term is much less than unity; i.e., in the neighborhood of the center of the pass band the oscillations of the attenuation characteristic are negligible, and the filter is essentially freely passing.

3. *Attenuation Region*

Outside the range $-2\leqq\rho\leqq 2$, θ is a complex quantity.

$$\theta=\pm j\ln\left\{\frac{\rho}{2}+\left[\left(\frac{\rho}{2}\right)^2-1\right]^{\frac{1}{2}}\right\}. \quad (45)$$

Let

$$\xi=\left\{\frac{\rho}{2}+\left[\left(\frac{\rho}{2}\right)^2-1\right]^{\frac{1}{2}}\right\}$$

and

$$\psi=\ln\xi.$$

Then for $-\infty<\frac{\rho}{2}<-1$,

$$\theta=\pm j(\ln\xi\pm j\pi)=\pm\pi\pm j\psi, \quad (46A)$$

and for $1<\frac{\rho}{2}<\infty$,

$$\theta=\pm j\ln\xi=\pm j\psi. \quad (47A)$$

Then (22) becomes

$$\beta=\frac{1}{2}+\frac{1}{4m^2}\left[\frac{\sinh(n-1)\psi}{\sinh\psi}\right]^2+\frac{1}{2}\left[\frac{\sinh n\psi}{\sinh\psi}\right]^2+\frac{m^2}{4}\left[\frac{\sinh(n+1)\psi}{\sinh\psi}\right]^2. \quad (48)$$

At points greater than about $(3/n^2)$ half-bandwidths away from "cut off," the hyperbolic sines and cosines in the numerators of the fractions are well approximated by exponentials; (48) expressed in decibels reduces to the approximate form

$$A\approx 8.7n\psi+20\log_{10}[\cosh(\psi+\nu)]-20\log_{10}\sinh\psi-6, \quad (49)$$

where A = attenuation in decibels $=10\log_{10}\beta$,
$\nu=\ln m=2.3\log_{10}m$,

At points greater than about 5 half-bandwidths away from "cut off," the formula simplifies further to

$$A\approx 8.7n\psi+8.7\nu-6. \quad (50)$$

Finally, if for m we write the optimal value (26), then for large n

$$A\approx 8.7n\psi-\frac{8.7}{n}-6. \quad (51)$$

4. *Conclusions*

An expression, simple in appearance, has been derived for a special class of resonant band-pass filters designed for ease of construction and adjustment. It turns out that this expression is more readily evaluated analytically when the number of filter sections is large. For that case, filters that behave well except at the edges of the pass band are readily designed. The undesirable behavior in these neighborhoods is readily traceable to a factor $\sin^2\theta$ appearing in the expression for the attenuation characteristic; this conclusion suggests that further work might proceed in the direction of reducing the variation in this factor.

Elements in the Design of Conventional Filters

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VARIOUS elements of practical interest in the design of conventional multisection filters are discussed. The first part of the paper deals with the graphical calculation of attenuation and phase using templates developed by T. Laurent and E. Rumpelt.[1] Starting from Rumpelt's results and using his notation, further applications are described, particularly in connection with Cauer's method of determining optimum values of filter parameters.

In the second part, a method of calculating insertion loss and phase is proposed. The method is based on an extension of various formulas presented by R. Feldtkeller[2] and gives a better understanding than do classical formulas of the behavior of insertion loss.

The third part is devoted to dissipative losses in filters. The accuracy of known formulas is discussed and more rigorous expressions are proposed.

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In the classical method of filter design, the calculation of individual filter sections and their combination is fairly simple but the determination of their parameters to suit given over-all requirements often necessitates several cut-and-try processes. The topics discussed in this paper have been selected according to their importance in shortening these processes. Some formulas are new but many more or less familiar results are included for the sake of completeness and to permit easy reference. The publication of an integrating paper of this kind as an addition to existing text books seemed desirable.

No apology is made for expressing attenuation in nepers (1 neper = 8.686 decibels) and phase in radians (1 radian = 57.3 degrees), even in graphs. The use of more practical units is unsatisfactory in theoretical formulas and inconsistency in notation would make cross references difficult. Natural logarithms are used throughout the paper.

1. Calculation of Image Attenuation and Phase

After a description of Rumpelt's method for a low-pass filter (Section 1.1), template construction is used to find the optimum allocation of attenuation poles (Section 1.2). Similar problems for band-pass filters are considered in Section 1.3. A final section deals with the graphical calculation of phase shift, its knowledge being necessary to permit an easy computation of the insertion loss.

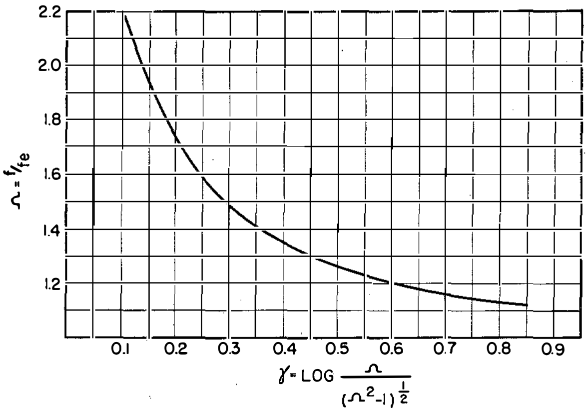


Fig. 1—Frequency transformation for a low-pass filter in the attenuation range.

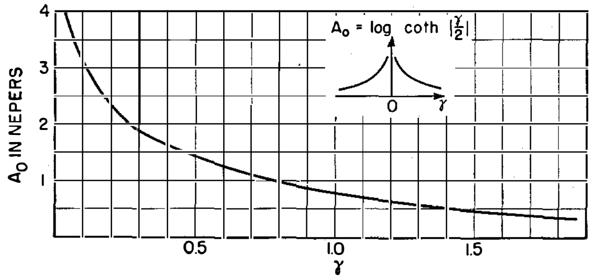


Fig. 2—Attenuation template.

[1] E. Rumpelt, "Schablonenverfahren für den Entwurf elektrischen Wellenfilter auf der Grundlage der Wellenparameter," *Telegraphen- Fernsprech- Funk- und Fernsehtechnik,* v. 38, pp. 203–210; August, 1942. An English description of the template method is given by F. Scowen, "An Introduction to the Theory and Design of Electrical Wave Filters," Chapman & Hall, London, 1945; Chapter 14.

[2] R. Feldtkeller, "Einführung in die Vierpoltheorie," 3d Edition, Hirzel, Leipzig, 1943.

1.1 Image Attenuation of Low-Pass Filters

The template method for a low-pass filter having a cut-off frequency f_c is based on the transformation of the frequency scale defined by

$$\gamma = \tfrac{1}{2} \log \frac{\Omega^2}{\Omega^2 - 1}\,; \quad \Omega = \frac{f}{f_c}, \tag{1}$$

which brings the attenuation range (f_c, ∞) of f into the interval (∞, 0) of γ as shown on Fig. 1.

The image attenuation of a constant-k full section, giving a pole of attenuation at $\gamma = 0$ is

$$A_o(\gamma) = \log \coth |\gamma/2| \tag{2}$$

and is shown in Fig. 2.

The image attenuation of a filter composed of n sections having attenuation poles at $f_1, f_2 \ldots f_n$, respectively, is

$$A(\gamma) = A_o(\gamma - \gamma_1) + A_o(\gamma - \gamma_2) + \ldots + A_o(\gamma - \gamma_n), \tag{3}$$

where $\gamma_1, \gamma_2 \ldots \gamma_n$ are the transformed frequencies corresponding to $f_1, f_2 \ldots f_n$ by (1). Each transformed pole γ_i, where $i = 1, 2 \ldots n$, is related to Zobel's parameter m_i of the corresponding section by

$$\gamma_i = -\log m_i. \tag{4}$$

From (3), and using a template corresponding to (2), the graphical construction is obvious and is illustrated in Figs. 3A and 3B, where the image attenuation of a three-section filter is shown both on normal and transformed frequency scales (curves A). For filters containing half-sections, half-templates should be used.

The preceding results are proved in Rumpelt's paper but an alternative derivation based on Bode's relations between image attenuation and phase will be outlined. If the attenuation of any minimum-phase-shift network is specified in some interval on the real frequency range and if the phase is specified in the complementary interval, one of Bode's relations[3] permits the determination of the complete transmission characteristics. In the case of a filter, the image attenuation is known to be zero in the transmission range and the image phase is necessarily a multiple of $\pi/2$ in the attenuation range. The only possible variation of phase in this range is thus a number of jumps from one multiple of $\pi/2$ to another; to each jump of $\pi/2$ at some frequency f_i there is a corresponding attenuation given by Bode's weighting function $\frac{1}{2} \log \coth \left| \frac{\gamma - \gamma_i}{2} \right|$ equivalent to the attenuation of a half section having a pole at f_i, and the image attenuation of any filter is thus the sum of a number of similar functions with different values of γ_i.

A reciprocity theorem is immediately deduced from the graphical construction: the image attenuation at frequency f_1 produced by a section having a pole at f_2 is equal to the attenuation at f_2 produced by a section having a pole at f_1. On the normal frequency scale, this propriety is far from being obvious.

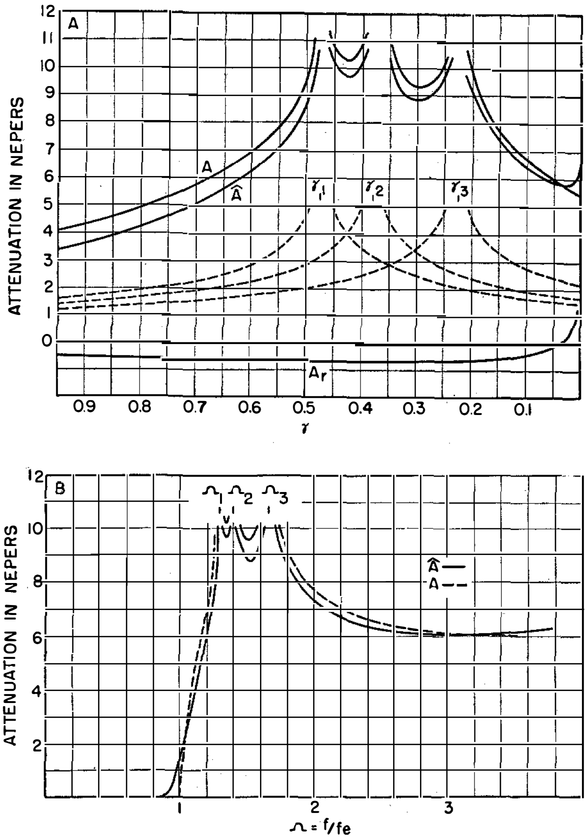


Fig. 3—Image attenuation A and effective attenuation $\hat{A}$ of a 3-section low-pass filter in the attenuation range. In 3A, curve A is constructed by means of templates and $\hat{A}$ includes the reflection loss A_r, 3B shows the attenuation characteristics plotted against natural frequency. $\hat{A}$ is the solid line and A is dashed.

[3] H. W. Bode, "Network Analysis and Feedback Amplifier Design," 1st Edition, Van Norstrand, New York, 1945; see equation (14–33), p. 329.

Sections with $m > 1$ are possible in lattice configurations. The template method can be used for such sections: the associated attenuation pole lies in the negative part of the γ scale, corresponding to an imaginary frequency. Such a section gives at any real frequency a lower attenuation than a constant-k section.

1.2 Attenuation for a Given Range and Number of Sections

The following problem often arises in practical design: given the cut-off frequency f_c of a low-pass filter, what is the minimum number of sections required to secure an image attenuation larger than a prescribed minimum value A_{min} over a given interval (f_a, f_b) of the attenuation range; or, conversely, what is the maximum attenuation obtainable in a given range with a given number of sections? To obtain the required attenuation with the minimum number of sections, all the attenuation poles must, of course, be different and suitably distributed over the interval (f_a, f_b).

If the interval (f_a, f_b) is brought into the interval (γ_a, γ_b) of the transformed frequency scale, the solution of the problem is seen to depend only on the magnitude $\Delta = \gamma_a - \gamma_b$ of the transformed interval and not on its absolute allocation on the scale. The first part of the solution consists in finding the relations between the required attenuation A_{min}, the interval Δ, and the number of sections n. These relations are expressed in graphical form in Fig. 4.

In the case of a single-section filter ($n=1$), the solution is fairly obvious: the pole is in the midpoint of the interval and the attenuation everywhere in the interval is larger than the value at both edges; thus

$$A_{min} = A_o(\tfrac{1}{2}\Delta). \quad (5)$$

For two or more sections, the solution can be found by cut-and-try methods, the general shape of the attenuation characteristic being such that $(n-1)$ equal minima occur between the n poles. The value A_{min} is the common value of the minima and is equal to the attenuation at the limits of the interval. Such a characteristic is said to have Chebishev* behavior in the interval and is shown in Fig. 5 for the case $n=2$ and $\Delta = 1$.

* This name is spelled variously in English, commonly as "Tchebyscheff".

The exact mathematical solution depends on elliptic functions and has been obtained by W. Cauer.[4] As Cauer's results are described with

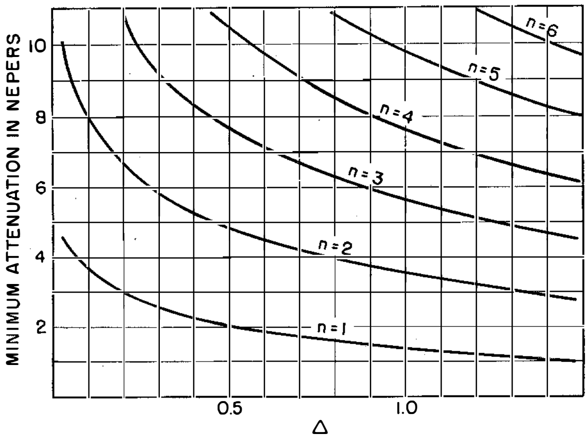


Fig. 4—Image attenuation A_{min} obtainable in an interval Δ of the γ scale with n sections and suitably located poles.

reference to lattice filters, the fact that they are of immediate practical value for conventional ladder-type multisection filters has often been overlooked; the text books of A. T. Starr[5] and E. A. Guillemin[6] only mention this method in chapters devoted to lattice filters and the known practical limitations of the lattice structure have prevented the widespread acceptance of Cauer's results.

It will first be shown how the curves of Fig. 4 have been obtained. They can be deduced from Cauer's formulas, but much shorter calculations are possible if a modular transformation of elliptic functions is applied as follows: from given values of Δ and n, the angle $\alpha = \cos^{-1}(e^{-\Delta})$ is computed and the corresponding modular constant q, or its logarithm, is read in a table of elliptic functions (in most tables[7] $\log_{10} q$ is tabulated versus α); from $\log_{10} q_n = n \log_{10} q$, a new modular constant q_n is deduced and, from the corresponding angle α_n obtained in the table, the attenuation A_{min} is calculated by the formula

$$A_{min} = 2 \tanh^{-1} (\cos \alpha_n)^{\frac{1}{2}}. \quad (6)$$

[4] W. Cauer, "Siebschaltungen," VDI Verlag, Berlin, 1931.
[5] A. T. Starr, "Electrical Circuits and Wave Filters," 2nd Edition, Pitman, London, 1938.
[6] E. A. Guillemin, "Communication Networks," Volume 2, 1st Edition, J. Wiley and Sons, New York, 1935.
[7] See for instance, E. Jahnke and F. Emde, "Tables of Functions," 4th Edition, Dover, New York 1945; p. 49.

According to S. Darlington,[8] the possibility of simplifying Cauer's formulas by means of a modular transformation was first suggested by E. L. Norton but no published account of this, for the case considered here, is known to the author. The preceding formulas are relatively easily deduced from Cauer's known formulas and it seems unnecessary to give a detailed account of the proof. Using approximate formulas for the elliptic functions, it can be proved that the attenuation increases nearly proportionally to the number of sections except for very small values of Δ.

The second part of the problem is to determine the allocation of the attenuation poles in the interval Δ to have equal intermediate minima.

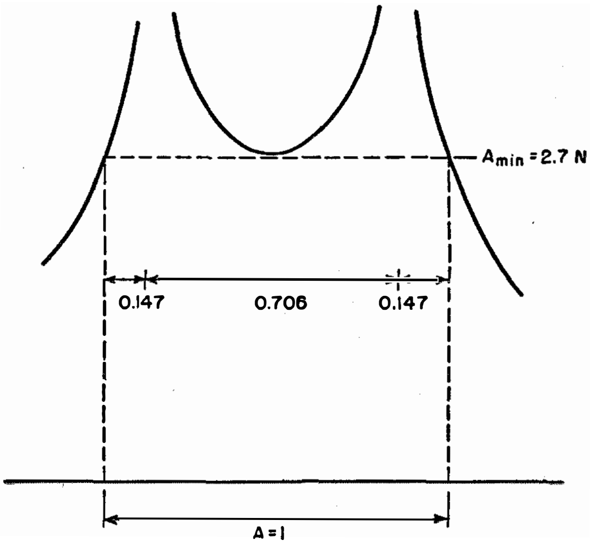


Fig. 5—Template construction for a two-section filter with Chebishev behavior.

The results will first be described for the case $n=2$ with reference to the example of Fig. 5. It appears that for $\Delta=1$, the distribution of the poles is specified by the numbers 0.147-0.706-0.147. If the interval is $\Delta\neq 1$, the same relative distribution remains approximately correct, i.e., the successive intervals become 0.147Δ-0.706Δ-0.147Δ. Only for very small values of Δ does the distribution depart from this simple rule, i.e., the ratios of the partial intervals to the total interval become themselves functions of Δ. Table 1 gives the relative values of the partial intervals up to the case where $n=6$. As already mentioned, these

TABLE 1

RELATIVE VALUES OF PARTIAL INTERVALS

Number of Sections	Relative Distribution of Poles
1	0.5-0.5
2*	0.147-0.706-0.147
3	0.067-0.433-0.433-0.067
4	0.048-0.261-0.382-0.261-0.048
5	0.025-0.156-0.319-0.319-0.156-0.025
6	0.017-0.130-0.223-0.260-0.223-0.130-0.017

* See Fig. 5.

figures are only approximate and have been obtained from Cauer's formulas by replacing the elliptic functions by the first term of their expansion for small values of the modulus. They are sufficiently accurate for all practical purposes, except for such cases where a very high attenuation is required in a quite narrow interval. The approximate distribution of the poles is given by the following rule: for n poles, the distance between the kth and the $(k+1)$th pole is $\Delta \sin(\pi/2n) \sin(k\pi/2n)$ and the distance between the ends of the Δ interval and the first or last pole is $\Delta \sin^2 \pi/4n$.

1.3 Image Attenuation of Band-Pass Filters

Band-pass filters having a symmetrical characteristic with respect to the geometrical midband frequency when plotted on a logarithmic frequency scale can be reduced to equivalent low-pass filters by means of the well-known transformation

$$\Omega=\frac{f^2-f_o^2}{f(f_c-f_{-c})}, \tag{7}$$

where the following notations are used.

$$f_c,\ f_{-c}=\text{cut-off frequencies } (f_{-c}<f_c),$$
$$f_o=(f_{-c}f_c)^{\frac{1}{2}} \text{ midband frequency.}$$

In the general dissymmetrical case, a different transformation must be used, which, unlike (7), has no physical counterpart. It is defined by

$$\left.\begin{aligned}\gamma&=\gamma_\infty+\tfrac{1}{2}\log\frac{f^2-f_{-c}^2}{f^2-f_c^2},\\ \gamma_\infty&=\tfrac{1}{2}\log(f_c/f_{-c}).\end{aligned}\right\} \tag{8}$$

[8] S. Darlington, "Synthesis of Reactance 4-Poles," *Journal of Mathematics and Physics*, p. 329; September, 1939.

The curve γ versus f is not unique as in the low-pass case but depends on the bandwidth. When a complete curve is required, it is simpler to invert the transformation (8) and use

$$f=f_o\left[\frac{\sinh\,(\gamma+\gamma_\infty)}{\sinh\,(\gamma-\gamma_\infty)}\right]^{\frac{1}{2}}. \tag{9}$$

The transformations (8) bring the upper attenuation range (f_c, ∞) into the positive interval (∞, γ_∞) on the γ scale and the lower attenuation range $(0, f_{-c})$ into the negative interval $(-\gamma_\infty, -\infty)$. The interval $(-\gamma_\infty, \gamma_\infty)$ does not correspond to any real frequency and decreases with the relative bandwidth. The transformation is symmetrical, which means that to two frequencies f_1 and f_2, such that $f_1f_2=f_o^2$, correspond two opposite values of γ, i.e., $\gamma_2=-\gamma_1$. The image attenuation of a composite filter is still computed by means of (3) but the function $A_o(\gamma)$, which has a pole at $\gamma=0$ in the imaginary frequency range, corresponds to a filter that cannot be realized as a ladder section. It should be noted that to any imaginary frequency there is a corresponding real γ in the interval $(-\gamma_\infty, \gamma_\infty)$ and the point $\gamma=0$ corresponds to $f=jf_o$. Any filter with a pole in this interval can be realized as a lattice section.

For a narrow-band-pass filter, the interval $(-\gamma_\infty, \gamma_\infty)$ is quite small and the template construction shows that a filter having a pole at both zero and infinite frequencies (i.e., at $-\gamma_\infty$ and γ_∞) or a filter having a double pole at the infinite frequency (i.e., at γ_∞) are nearly equivalent at any finite frequency. As a consequence, an entire section of the so-called "three-element type" has practically the same attenuation as a constant-k half section.

When frequency-symmetrical filters are being designed, either the transformation (8) can be used or the filter can first be reduced to the

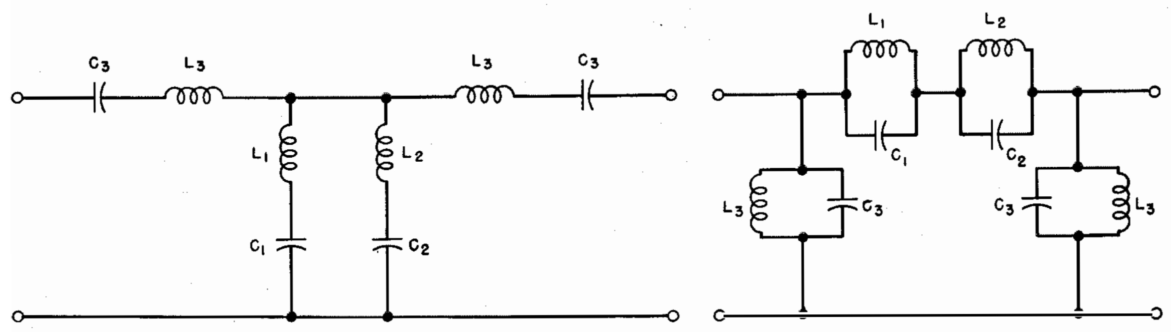


Fig. 6—Design formulas of dissymmetrical band-pass T and π sections.

$\frac{L_1}{L_0}$	$\sinh\,(\gamma_\infty-\gamma_1)\tanh\frac{\gamma_2-\gamma_1}{2}$	$\frac{C_1}{C_0'}$
$\frac{L_2}{L_0}$	$\sinh\,(\gamma_2-\gamma_\infty)\tanh\frac{\gamma_2-\gamma_1}{2}$	$\frac{C_2}{C_0'}$
$\frac{L_3}{L_0}$	$\dfrac{\cosh\left(\gamma_\infty-\frac{\gamma_1+\gamma_2}{2}\right)}{\cosh\frac{\gamma_2-\gamma_1}{2}}$	$\frac{C_3}{C_0'}$
$\frac{C_0}{C_1}$	$\sinh\,(\gamma_0-\gamma_1)\tanh\frac{\gamma_2-\gamma_1}{2}$	$\frac{L_0'}{L_1}$
$\frac{C_0}{C_2}$	$\sinh\,(\gamma_2-\gamma_0)\tanh\frac{\gamma_2-\gamma_1}{2}$	$\frac{L_0'}{L_2}$
$\frac{C_0}{C_3}$	$\dfrac{\cosh\left(\gamma_0-\frac{\gamma_1+\gamma_2}{2}\right)}{\cosh\frac{\gamma_2-\gamma_1}{2}}$	$\frac{L_0'}{L_3}$
$\frac{L_0}{R}$	$\frac{1}{2\pi(f_c-f_{-c})}$	$C_0'R$
C_0R	$\frac{f_c-f_{-c}}{2\pi f_cf_{-c}}$	$\frac{L_0'}{R}$

γ_1 is negative and corresponds to an attenuation pole f_1 (resonance L_1C_1) in the lower attenuation range.

γ_2 is positive and corresponds to an attenuation pole f_2 (resonance L_2C_2) in the upper attenuation range.

γ_0 corresponds to zero frequency and is equal to $-\gamma_\infty$.

equivalent low-pass filter by (7) and then transformation (1) applied. The two methods give different f-versus-γ relations because, in the second one, the upper and lower attenuation ranges of the band-pass are confluent into the single attenuation range of the low-pass. Denoting by γ and γ', respectively, the transformed frequencies corresponding to the same f by the two methods, the relation between γ and γ' is

$$\gamma' = \log \cosh \gamma - \log \cosh \gamma_\infty \qquad (10)$$

and is double valued in γ, two symmetrical values $+\gamma$ and $-\gamma$ corresponding to the same γ'. This relation may be useful when a dissymmetrical filter contains some symmetrical sections.

A great simplification is obtained in the design formulas of the general dissymmetrical band-pass sections by directly expressing the values of the components in terms of the transformed frequencies. The most general T and π sections with constant-k image impedances are shown in Fig. 6 and have two attenuation poles f_1 and f_2 arbitrarily located, the first in the lower and the second in the upper attenuation range. The corresponding transformed poles by (10) are γ_1 (negative) and γ_2 (positive). The design formulas are also given in Fig. 6, where R denotes the nominal impedance (image impedance at f_o). Formulas for all other types of band-pass filters can be obtained as particular cases of these.

The problem of designing a band-pass filter meeting independent attenuation requirements in both attenuation ranges leads to a template construction with two Δ intervals. The results of Section 1.2 are of immediate use only for a single interval with uniform minimum attenuation and can be applied to band-pass filters in three cases: (A) for symmetrical filters and symmetrical attenuation requirements, both Δ intervals are transformed into the single Δ interval of the equivalent low-pass filter, (B) for narrow-band-pass filters with a common value of the required attenuation extending to zero and infinite frequencies, the interval $(-\gamma_\infty, \gamma_\infty)$ separating the two Δ intervals is negligible, and (C) for any band-pass filters with attenuation requirements in only one of the attenuation ranges.

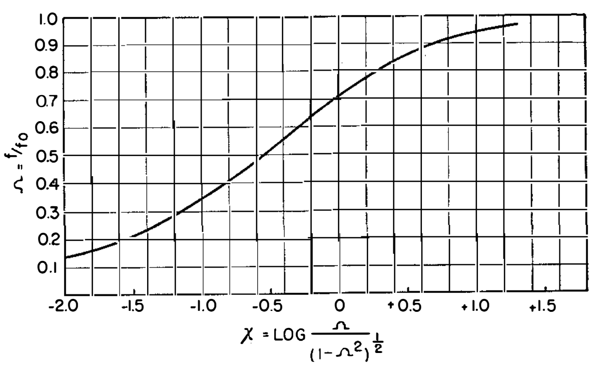


Fig. 7—Frequency transformation for a low-pass filter in the transmission range.

In this last case, a low-pass or a high-pass filter should be sufficient but a band-pass is often preferred when some attenuation is desirable in the other range. In some cases, the band-pass solution may even be more economical as is evident from the following: let the upper cut-off be fixed at f_c and suppose a given attenuation is required in a part (f_a, f_b) of the upper range $(f_b > f_a > f_c)$; taking a band-pass filter with the lower cut-off at f_{-c} instead of a low-pass filter produces a shortening of

$$\tfrac{1}{2} \log \frac{f_a^2(f_b^2 - f_c^2)}{f_b^2(f_a^2 - f_{-c}^2)}$$

of the interval Δ corresponding to (f_a, f_b); this may reduce the number of sections necessary to cover the required attenuation. Band-pass sections are not more expensive than low-pass sections if the unnecessary coils are omitted by means of impedance transformations and the terminal shunt coils are used as transformers.[9] In addition, the image impedances will give a better match in the band-pass case.

1.4 Image Phase-Shift

Calculation of the image phase in the transmission range of a low-pass filter is based on a frequency transformation complementary to (1), defined by

$$\chi = \tfrac{1}{2} \log \frac{\Omega^2}{1-\Omega^2}\,; \quad \Omega = \frac{f}{f_c}\,, \qquad (11)$$

which brings the transmission range $(0, f_c)$ into the whole real interval $(-\infty, \infty)$ of χ, the point $\chi = 0$ corresponding to $\Omega = \tfrac{1}{2}2^{\frac{1}{2}} = 0.71$ (Fig. 7). The image phase of a constant-k section is

$$B_o(\chi) = \cot^{-1} \sinh(-\chi) = -2 \cot^{-1} e^{\chi} \qquad (12)$$

[9] V. Belevitch, "Extension of Norton's Impedance Transformation to Band-pass Filters." *Electrical Communication*, v. 24, pp. 59–65; March, 1947.

and the image phase of a filter composed of sections having attenuation poles at the points $\gamma_1, \gamma_2 \ldots$ of the γ scale is

$$B(\chi) = B_o(\chi - \gamma_1) + B_o(\chi - \gamma_2) + \ldots \quad (13)$$

The curve B_o is shown in Fig. 8 and the template construction results from (13). The image phase

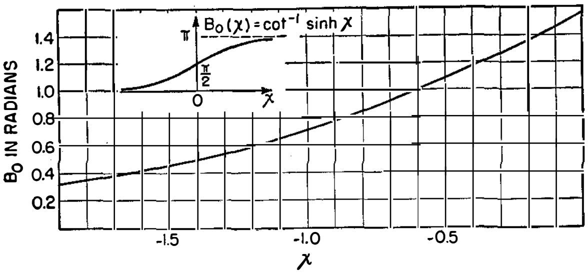


Fig. 8—Phase template.

increases from 0 at zero frequency to the value $n\pi$, where n is the number of entire sections. In the attenuation range, the image phase decreases by π at each attenuation pole, or by $\pi/2$ at a half pole (corresponding to a half section). The template construction shows that the slope of the phase of a section with a finite attenuation pole is always larger than the slope corresponding to a constant-k section ($\gamma = 0$); this slope can only be decreased by using a section with a pole at a negative γ, i.e., at an imaginary frequency.

The template construction of the image phase is illustrated in Fig. 9A for the filter discussed in connection with Fig. 3, and the image phase is shown on a natural-frequency scale in Fig. 9B (curve B).

For dissymmetrical band-pass filters, the transformation is

$$\chi = \gamma_\infty + \tfrac{1}{2} \log \frac{f^2 - f_{-c}^2}{f_c^2 - f^2}, \quad (14)$$

or inverted

$$f = f_o \left[\frac{\cosh (\chi + \chi_\infty)}{\cosh (\chi - \gamma_\infty)} \right]^{\frac{1}{2}} \quad (15)$$

and transforms the frequencies f_{-c}, f_o, f_c into $\chi = -\infty, 0, \infty$.

2. *Calculation of Effective Attenuation and Phase*

When a filter is working between a receiving impedance R_2 and a generator having an impedance R_1 and producing an electromotive force E, its actual loss and phase differ from its image attenuation and phase because of the mismatch between the filter image impedances and the terminal resistances. The effective attenuation $\hat{A}$ and the effective phase $\hat{B}$ will be defined by

$$\hat{A} + j\hat{B} = \tfrac{1}{2} \log \frac{P_o}{P_r},$$

where P_r is the complex power received in R_2 through the filter and P_o the maximum power $E^2/4R_1$ available from the generator. The notion of insertion loss, which is more commonly used in English and American text books, differs from the effective attenuation defined here by the reflection loss between R_1 and R_2. The effective attenuation is more appropriate to deal with passive networks because it always gives a positive loss, whereas a passive network between unequal impedances can give an insertion gain. For symmetrical terminations, both notions coincide. The notion of effective attenuation has been introduced by some authors under a different name; the term adopted here is quoted as a translation of the widely used German term "Betriebsdämpfung" in the Comité Consultatif International Téléphonique vocabulary.[10] The effective phase and the insertion phase always coincide for networks operating between pure resistances.

In the next section, the behavior of the effective attenuation in the transmission range is analyzed. The attenuation is expressed as a function of the image phase (already discussed in Section 1.4) and of the image impedances. Sections 2.2 and 2.3 are devoted to the study of the image impedances of low-pass and band-pass filters, respectively, and to the related questions of optimum matching. Section 2.4 contains a discussion of effective attenuation and phase in the attenuation range and a final section consists of some remarks on the return loss.

2.1 Effective Attenuation and Phase in Transmission Range

The effective attenuation and phase will be expressed in terms of image parameters. Since the image impedances W_1 and W_2 enter in the formula only through their ratios to the terminal

[10] See Comité Consultatif International Téléphonique, Tenth Plenary Meeting, Budapest, 1934; English edition issued by International Standard Electric Corporation, London, 1936; p. 169.

resistances, it is convenient to introduce normalized image impedances

$$w_1=\frac{W_1}{R_1} \quad \text{and} \quad w_2=\frac{W_2}{R_2}.$$

The following expressions hold in the transmission range for a filter working between pure resistances.

$$\hat{A}=\tfrac{1}{2}\log\left[1+\frac{(1-w_1w_2)^2}{4w_1w_2}\sin^2 B + \frac{(w_1-w_2)^2}{4w_1w_2}\cos^2 B\right], \quad (16)$$

$$\tan\hat{B}=\frac{1+w_1w_2}{w_1+w_2}\tan B. \quad (17)$$

Two particular cases will first be examined; one for *symmetrical filters* where $w_1=w_2$, i.e., $W_1/R_1=W_2/R_2$, and the other for *antisymmetrical filters* where $w_1w_2=1$, i.e., $W_1W_2=R_1R_2$. In the first case, (16) becomes

$$\hat{A}=\tfrac{1}{2}\log\left[1+\left(\frac{w^2-1}{2w}\sin B\right)^2\right], \quad (18)$$

where w stands either for w_1 or w_2, and in the second case a similar formula is obtained but with cos B replacing sin B. The formulas in those two cases and the corresponding formulas for the phase have been obtained by R. Feldtkeller;[11] (16) and (17) are easily obtained as extensions of these. A short derivation of (18) is given in the appendix, Section 4.

From (18), it is obvious that

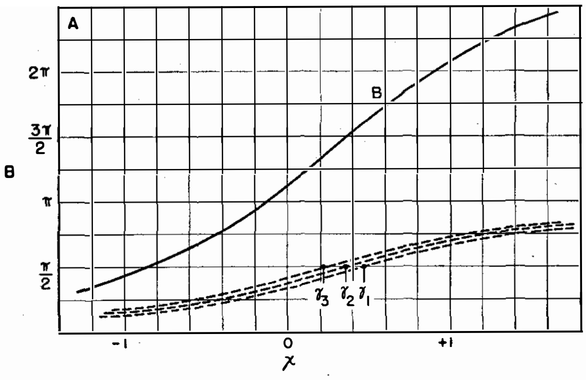
$$\hat{A}\leqslant\tfrac{1}{2}\log\left[1+\left(\frac{w^2-1}{2w}\right)^2\right]=\log\frac{w^2+1}{2w}, \quad (19)$$

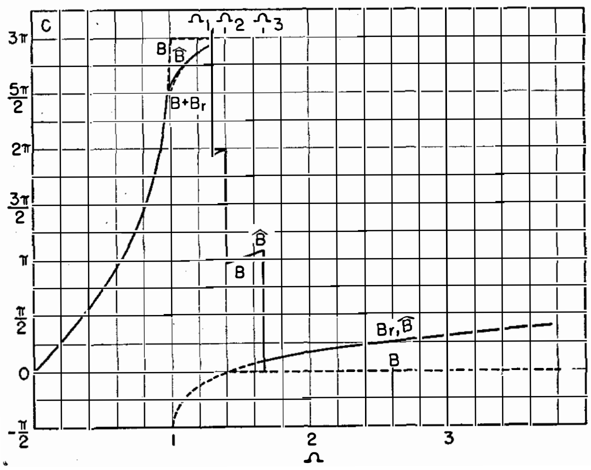
so that the curve of the effective attenuation oscillates between zero and the envelope given by the last member of (19); for a symmetrical filter, $\hat{A}$ is zero for frequencies at which $B=k\pi$ and the curve touches the envelope at frequencies for which $B=(k+\frac{1}{2})\pi$; for antisymmetrical filters, the reverse situation is true.

Introducing the notation

$$A_{\text{env}}=\log\frac{w^2+1}{2w}, \quad (20)$$

[11] See footnote reference 2, p. 84.





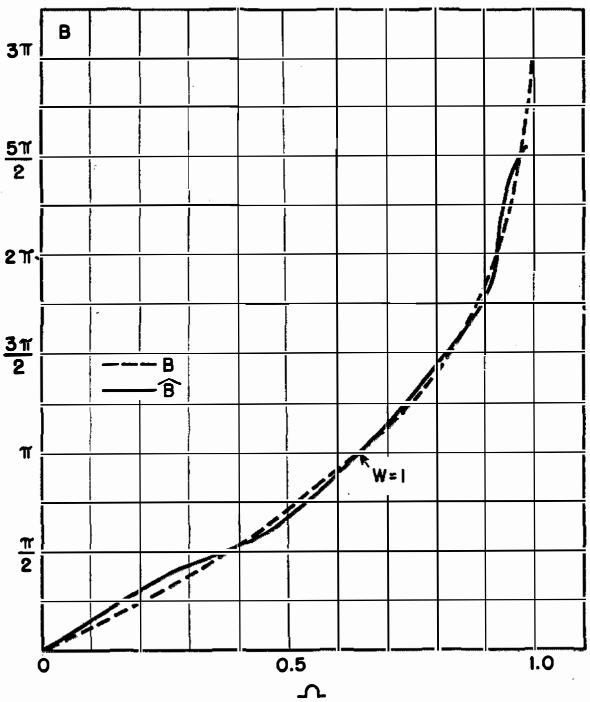


Fig. 9—Image phase B and effective phase $\hat{B}$ of a 3-section low-pass filter. A. Template construction of B. B. Phase characteristics on natural-frequency scale. The difference between $\hat{B}$ (dashed line) and B (solid line) is magnified 5 times. C. Construction of the effective phase in the attenuation range by adding the reflection phase B_r.

(19) gives $\hat{A} \leqslant A_{env}$ and permits a rapid and somewhat large estimation of $\hat{A}$ at any frequency without calculating the image phase. It should be noted that A_{env} is neither the reflection loss between the image impedance and the terminal resistance nor twice this reflection loss, but is actually the reflection loss corresponding to twice the mismatch ratio. This is easily explained with reference to the known fact that the input impedance of a filter of image impedance W terminated on R at the output can take any value from R to W^2/R, depending on the image phase, the largest mismatch being thus in the ratio $W^2/R^2 = w^2$.

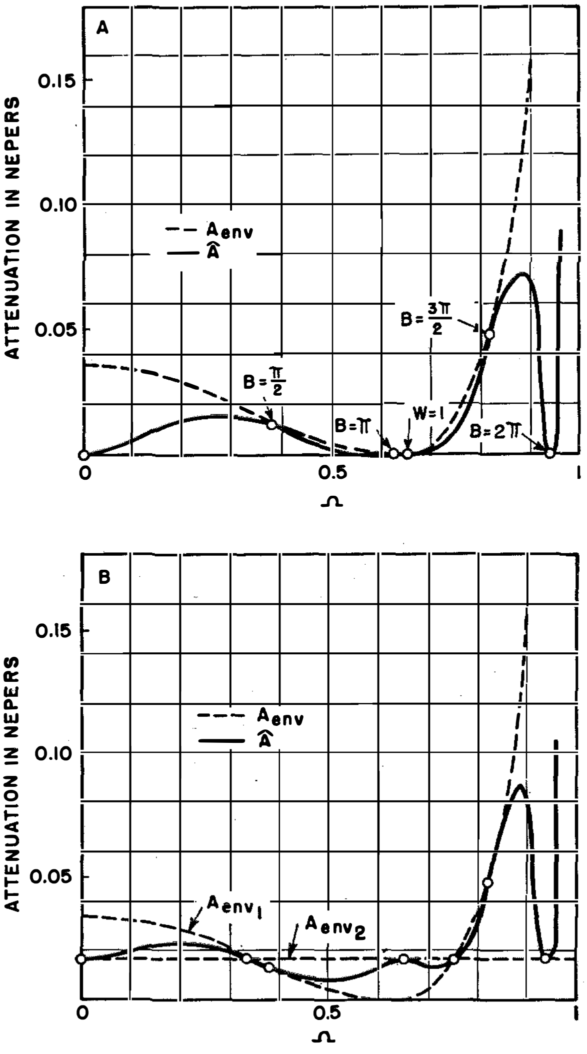


Fig. 10—Effective attenuation in the transmission range of a 3-section low-pass filter, the dashed lines being A_{env} and the solid lines $\hat{A}$. A. Between equal terminations. B. Between unequal terminations.

The oscillatory behavior of the effective attenuation in the transmission range of a low-pass filter is illustrated in Fig. 10A, where the dotted curve represents A_{env}. Calculations were based on the image-phase characteristic of Fig. 9 and on the assumption of constant-k image impedances of the midshunt type, the filter design impedance being 77 percent of the terminal resistances, which gives optimum matching over 80 percent of the transmission range. In this example, the zero-loss point due to $B = \pi$ and the zero-loss point due to perfect matching nearly coincide.

For filters neither symmetrical nor antisymmetrical, the general formula (16) shows that $\hat{A}$ must have a value intermediate between A_{env_1} and A_{env_2} where

$$\left.\begin{aligned} A_{env_1} &= \log \frac{1+w_1w_2}{2(w_1w_2)^{\frac{1}{2}}}, \\ A_{env_2} &= \log \frac{w_1+w_2}{2(w_1w_2)^{\frac{1}{2}}}. \end{aligned}\right\} \qquad (21)$$

The first formula corresponds to the reflection loss between w_1 and $1/w_2$ and the second one to the reflection loss between w_1 and w_2. The example of Fig. 1 illustrates the oscillation of $\hat{A}$ between the two envelopes. The attenuation characteristic touches the second envelope at points where $B = k\pi$ and the first one when $B = (k+\frac{1}{2})\pi$. This results from the following approximate expression, which is equivalent to (16) for small attenuations,

$$A = A_{env_1} \sin^2 B + A_{env_2} \cos^2 B. \qquad (22)$$

In Fig. 10B, the same filter was used and only the terminal resistances differ. Denoting by R the common value of the terminal resistance in the first case, the terminations of the second case have been taken as $R_1 = \alpha R$ and $R_2 = R/\alpha$, with $\alpha = 0.83$. The filter itself is symmetrical in both cases and W is the image impedance. In the case of symmetrical terminations, A_{env} was given by (20) with $w = W/R$. For dissymmetrical terminations, $w_1 = W/\alpha R = w/\alpha$ and $w_2 = \alpha W/R = \alpha w$ so that $w_1w_2 = w^2$ and A_{env_1}, as given by the first of (21), coincides with A_{env} of the first case; A_{env_2} is reduced to a constant

$$A_{env_2} = \log \frac{1+\alpha^2}{2\alpha} = 0.017 \text{ neper.}$$

Comparison between Figs. 10A and 10B shows that no particular advantage can be obtained by matching differently the input and output of a filter. As the effective characteristics of symmetrical and antisymmetrical filters are much simpler to design, only such filters will be considered in the next paragraphs.

For symmetrical filters, the effective phase formula (17) becomes

$$\tan \hat{B} = \frac{1+w^2}{2w} \tan B = e^{A_{\text{env}}} \tan B, \qquad (23)$$

which is approximately equivalent to $\hat{B} = B + \frac{1}{2}A_{\text{env}} \sin 2B$ for small attenuations, so that $\hat{B}$ oscillates around B as shown in Fig. 9B. For antisymmetrical filters, (23) holds with cot instead of tan in both members.

At the cut-off frequencies, all formulas of this paragraph become indeterminate. Other methods of calculating the effective attenuation and phase were discussed in a previous paper.[12]

2.2 Optimum Matching for Low-Pass Filters

As a conclusion to the preceding paragraph, an upper limit of effective attenuation in the transmission range of a symmetrical or antisymmetrical filter can easily be estimated from the envelope attenuation given by (20), which depends on the image impedance alone. We will now investigate how the parameters of the image impedance must be chosen to obtain optimum matching conditions in a given part of the transmission range and calculate the corresponding maximum attenuation. This problem has been solved by E. A. Guillemin[13] in the most important cases and we will first summarize his results.

The normalized image impedance of the mid-series constant-k type is, for a low-pass filter,

$$w = \mu(1-\Omega^2)^{\frac{1}{2}}, \qquad (24)$$

where μ stands for the ratio W_o/R of the nominal impedance (image impedance at zero frequency) to the terminal resistance. To have the best possible matching from 0 to f_x, the mismatch ratio at frequency f_x must be $1/\mu$. Denoting by x the ratio f_x/f_c, this gives

$$\mu = (1-x^2)^{-\frac{1}{4}} \qquad (25)$$

and the corresponding maximum value of the envelope attenuation is

$$A_{\max} = \log \frac{1+\mu^2}{2\mu}. \qquad (26)$$

The values of μ and $A_{\max}$ are plotted versus x in Fig. 11A. The same results hold for the mid-shunt impedance but with μ replaced by $1/\mu$.

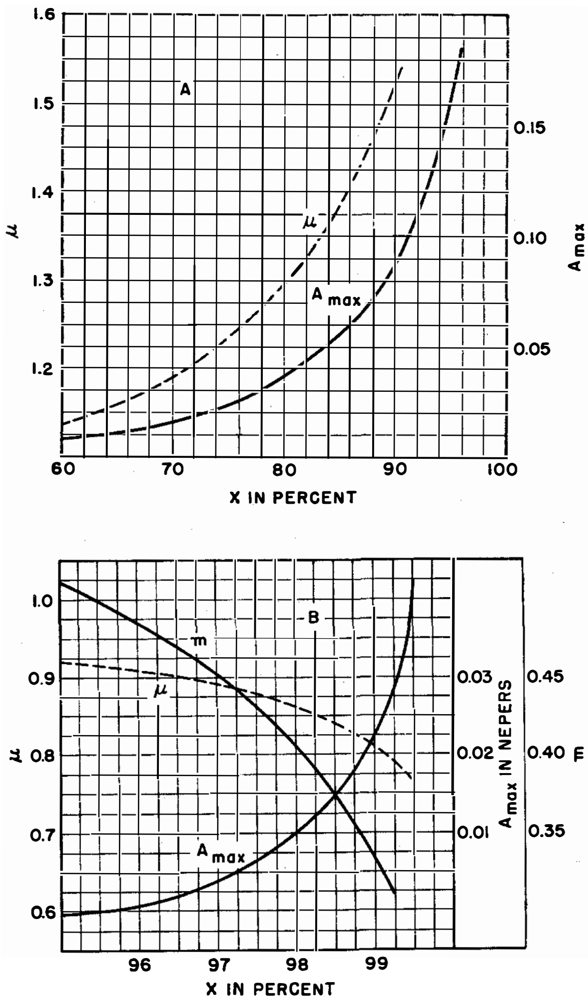


Fig. 11—Optimum image impedances and corresponding maximum attenuation in a given part x of the transmission range. A. Constant-k impedances. B. m-derived impedance. μ denotes the ratio of the nominal filter impedance to the terminating resistance for mid-series terminations; inverse ratio for mid-shunt terminations.

[12] V. Belevitch, "Insertion Loss and Effective Phase Shift in Composite Filters at Cut-off Frequencies," *Electrical Communication*, v. 24, pp. 192–194; June, 1947. For antisymmetrical filters the term "insertion loss" was incorrectly used in this paper and should be replaced by "effective attenuation."

[13] See footnote reference 6, pp. 313 and 333.

The normalized m-derived image impedance of the mid-series type is

$$w=\mu[1-(1-m^2)\Omega^2]^{-1}(1-\Omega^2)^{\frac{1}{2}}, \tag{27}$$

where μ has the same meaning as before and m is the usual parameter. The values of the parameters giving the optimum matching in the range $(0–f_x)$ are

$$\mu^{-2}=\tfrac{1}{2}[(1-x^2)^{\frac{1}{4}}+(1-x^2)^{-\frac{1}{4}}], \tag{28}$$

$$m^{-2}=1+(1-x^2)^{-\frac{1}{2}}, \tag{29}$$

and $A_{\max}$ is still given by (26). The results are represented in Fig. 11B. The value of m obtained from (29) automatically determines the location of the attenuation pole of the terminal half sections. If, from the attenuation point of view, a slightly different value is preferred, the resulting μ and $A_{\max}$ are modified. The nominal impedance must always be chosen so that the terminal resistance is the geometric mean between the largest and smallest value of the image impedance in the useful range.

A more general approach to the optimum matching problem will now be explained; it can be successfully applied to more intricate cases such as mm' impedances or when the useful range does not extend to zero frequency. Introducing the image return loss[14]

$$A_e=\log\frac{W+R}{W-R}=\log\frac{w+1}{w-1}, \tag{30}$$

which becomes infinite at frequencies where W and R are exactly matched, the condition $A\leqslant A_{\max}$ is transformed into

$$A\geqslant A_{e\min}=\log\coth\tfrac{1}{2}A_{\max}. \tag{31}$$

The condition $A_e\geqslant A_{e\min}$ is analogous to the condition $A\geqslant A_{\min}$ discussed in Section 1.2 and, to the optimum allocation of attenuation poles, corresponds now the allocation of the poles of A_e, or frequencies of exact matching. Constant-k and m-derived impedances have, respectively, one and two exact matching frequencies and their return-loss characteristics correspond to the image-attenuation characteristics of one- and two-section filters. The return-loss characteristic is computed with the help of the same template (Fig. 2) and the scale transformation is still (1) but with $\Omega=f_c/f$.

[14] The notation A_e was adopted for return loss because A_r will be used later for reflection loss; e is an abbreviation for echo.

We close this section by applying the results of Fig. 11B to the design of balancing networks for loaded cables. The mid-section impedance of a loaded cable is identical to the mid-shunt constant-k image impedance of a low-pass filter and can be simulated by a low-pass filter having the prescribed image impedance and terminated in a constant resistance at the other end. The effective input impedance of the filter will be close to the constant-k image impedance if the output image impedance approximates closely enough the terminal resistance. This is obtained by using an m-derived output image impedance and the whole balancing network consists of a half-section filter and its terminating resistance, which is in fact the network proposed by R. S. Hoyt.[15] The values of m and of the terminal resistance giving the best approximation in the interval $(0-f_x)$ are obtained from Fig. 11B and the degree of balance is expressed by the minimum return loss in the interval, given by (31), where $A_{\max}$ is obtained from Fig. 11B. In the particular case where $x=0.6$, corresponding to $m=0.67$ and $A_{e\min}=6.4$ nepers, the usual design formulas for an m-derived half section give numerical values for the network elements that coincide with Hoyt's values. This gives an easier proof of Hoyt's formulas and indicates that Hoyt's network gives the best balance up to 60 per cent of the cable cut-off frequency, which no longer corresponds to modern practice.

2.3 Optimum Matching for Band-Pass Filters

Band-pass image impedances symmetrical with respect to the mid-band frequency are obtained from low-pass impedances by transformation (7). If the approximation interval (f_{-x}, f_x) is symmetrically located in the transmission range so that

$$f_{-x}f_x=f_{-c}f_c=f_o^2, \tag{32}$$

the results of Figs. 11A and 11B can be used with

$$x=\frac{f_x-f_{-x}}{f_c-f_{-c}}. \tag{33}$$

In practical design, f_{-x}, f_x, and $A_{\max}$ are the data and x is deduced from Fig. 11; the theoretical cut-off frequencies are then obtained by solving

[15] R. S. Hoyt, "Impedance of Loaded Lines and Design of Simulating and Compensating Networks," *Bell System Technical Journal*, v. 3, pp. 414–467; July, 1924.

(32) and (33) in the following way:

$$\gamma_\infty = \sinh^{-1} \frac{f_x - f_{-x}}{2xf_o}; \quad f_{\pm c} = f_o\, e^{\pm \gamma_\infty}. \tag{34}$$

Band-pass image impedances that are not symmetrical with respect to the mid-band frequency

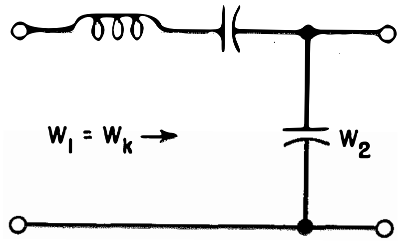


Fig. 12—Three-element band-pass half section.

are seldom used, but a particular case may be of interest. Consider the three-element half section shown in Fig. 12, having a constant-k input image impedance W_k, equal to R at the mid-band frequency, and the output image impedance[16]

$$W_2 = R\frac{f_c + f_{-c}}{f}\left(\frac{f^2 - f_{-c}^2}{f_c^2 - f^2}\right)^{\frac{1}{2}}. \tag{35}$$

This output impedance satisfies the relation

$$W_2(f)\, W_2(f_o^2/f) = R'^2, \tag{36}$$

where

$$R' = (1 + f_{-c}/f_c)R. \tag{37}$$

This means that, if the terminal output resistance is R', the mismatch ratio $w_2 = W_2/R'$ will take reciprocal values in opposite halves of the transmission range, and the envelope attenuation will be symmetrical. A filter composed of entire sections of the type considered and having image impedances W_2 terminated on R' at both ends, will have an envelope attenuation

$$A_{\text{env}} = \log \tfrac{1}{2}(w_2 + 1/w_2) \tag{38}$$

and

$$\tfrac{1}{2}(w_2 + 1/w_2) = (1 - f_{-c}/f_c)w_k + (1 + f_{-c}/f_c)w_k, \tag{39}$$

where

$$w_k = W_k/R = (1 - \Omega^2)^{-\frac{1}{2}} \tag{40}$$

and Ω stands for the transformed frequency (7). For narrow-band-pass filters, (37) gives $R' = 2R$ and, in (39), the first term can be neglected so that the envelope attenuation expressed in terms of the ratio x defined by (33) becomes

$$A_{\text{env}} = -\tfrac{1}{2}\log(1 - x^2), \tag{41}$$

which gives 0.35 nepers for $x = 0.7$ and shows that this type of termination is very poor.

2.4 Effective Attenuation and Phase in Attenuation Range

In the attenuation range, we calculate the effective attenuation by the usual composition formula. The interaction loss is negligible (less than (0.05 neper) as soon as the image attenuation is large (exceeds 1.5 neper). The effective attenuation is then

$$\hat{A} = A + A_{r1} + A_{r2}, \tag{42}$$

where A_{r1} and A_{r2} are the input and output reflection losses. The reflection loss between a purely imaginary image impedance $W = \pm jW'$ and an ohmic resistance R reaches its minimum value -0.345 neper for $W' = R$, is zero for $W'/R = 2 \pm 3^{\frac{1}{2}}$ and is positive for larger or smaller mismatch ratios. In any case, the inequality

$$\hat{A} \geqslant A - 0.69 \text{ neper} \tag{43}$$

holds, and the constant 0.69 neper must be added to the attenuation requirement before estimating the necessary number of sections from the curves of Fig. 4.

The exact expression of the reflection loss due to two constant-k image impedances, expressed as a function of the transformed frequency γ is

$$\begin{aligned} A_r &= A_{r1} + A_{r2} \\ &= \log\left[\mu(e^{2\gamma} - 1)^{\frac{1}{2}} + \mu^{-1}(e^{2\gamma} - 1)^{-\frac{1}{2}}\right] - 1.38. \end{aligned} \tag{44}$$

For $\mu = 1$, the minimum -0.69 is reached for $\gamma = 0.345$ ($\Omega = 1.41$) and A_r is zero for $\gamma = 1.29$ ($\Omega = 1.04$) and $\gamma = 0.04$ ($\Omega = 3.85$). Formula (44) has been used in the example of Fig. 3, which shows that A_r remains very close to its minimum value in the major part of the attenuation range. The only other important effect of the reflection loss is to produce infinite effective attenuation at infinite frequency.

The effective phase is calculated from a formula similar to (42) involving the image phase and the reflection phases, the interaction phase being neglected. For a symmetrical filter with normalized image impedances $\pm jw'$, this gives

$$\hat{B} = B + 2\tan^{-1} w' - \frac{\pi}{2}, \tag{45}$$

where the principal value (between 0 and $\pi/2$) of $\tan^{-1}$ must be considered. An example of the corresponding construction is shown in Fig. 9C.

[16] See footnote reference 5, p. 286.

2.5 Return Loss

We conclude this chapter by a few remarks stressing the analogy between the return loss in the transmission range and the effective attenuation in the attenuation range. One should first distinguish between the *image return loss* A_e already defined by (30) and the *effective return loss* $\hat{A}_e$ defined by a similar equation but involving the effective impedance $\hat{W}$ instead of the image impedance W; this impedance $\hat{W}$ is the value measured from one filter end when the other end is terminated by the terminal resistance. Input and output image return losses may be different, but the effective return loss has a unique value at both ends, even in the case of dissymmetrical terminations. This value is related to the effective attenuation by the formula

$$e \exp\left[-2\hat{A}\right] + e \exp\left[-2\hat{A}_e\right] = 1, \qquad (46)$$

by Feldtkeller,[17] resulting immediately from the conservation of energy in a nondissipative network. From this formula, any matching requirements imposed on a filter can be translated into attenuation requirements.

Like the effective attenuation, the effective return loss is the sum of a number of terms; image, reflection, and interaction return losses. The interaction return loss being negligible in the transmission range, a formula similar to (43) is obtained, i.e.,

$$\hat{A}_e \geqslant A_e - 0.69 \text{ neper.} \qquad (47)$$

3. *Dissipative Effects*

The dissipation in filter components produces two important effects: (A) a supplementary loss in the transmission range and (B) a rounding off of the attenuation poles.

The notations have been defined in Section 3.1, and a general formula for the transmission range (proved in the appendix, Section 4) is given. The orders of magnitude of the various terms are compared and, in Section 3.2, the discussion is further specialized to the case of low-pass filters. The corresponding discussion for band-pass filters is given in Section 3.3 and the behavior at the attenuation poles is described in Section 3.4.

3.1 General Formula for Transmission Range

The effect of dissipation in filters will be estimated with the help of the usual assumption that all inductors have the same Q, denoted by Q_L, and that all the capacitors have the same Q, denoted by Q_C. Usually Q_C is much larger than Q_L, so the effect of the dissipation in the capacitors is negligible, but the calculation is no more difficult if a finite value of Q_C is taken into account.

The Q factors only enter in the formulas through the following combinations

$$Q_s = \frac{2Q_CQ_L}{Q_C+Q_L}\,; \quad Q_d = \frac{2Q_CQ_L}{Q_C-Q_L}. \qquad (48)$$

If $Q_C \gg Q_L$, (48) becomes

$$Q_s = Q_d = 2Q_L. \qquad (49)$$

The following expression giving the additional effective attenuation $\delta\hat{A}$ due to dissipation in the transmission range of a symmetrical filter working between equal terminations is proved in the appendix.

$$\delta\hat{A} = \frac{1}{Q_s}\omega\frac{d\hat{B}}{d\omega} + \frac{1}{Q_d}(1-e^{-2\hat{A}})^{\frac{1}{2}} \cos \hat{B}, \quad \omega = 2\pi f. \qquad (50)$$

We will first show that the second term of (50) is usually negligible. Using the approximate value $A = A_{\text{env}} \sin^2 B$ and neglecting the difference between $\hat{B}$ and B, this second term is approximately $(1/Q_d)(\frac{1}{2}A_{\text{env}})^{\frac{1}{2}} \sin 2\hat{B}$. To compare the two terms, we will suppose that $Q_d = Q_s$ and write

$$\delta\hat{A} = \frac{1}{Q_s}\omega\frac{d\hat{B}}{d\omega}\left[1 + \frac{(\frac{1}{2}A_{\text{env}})^{\frac{1}{2}}}{d\hat{B}/d\omega}\frac{\sin 2\hat{B}}{\omega}\right]. \qquad (51)$$

The factor $(1/\omega) \sin 2\hat{B}$ is maximum or minimum when its derivative is zero, i.e., for $(1/\omega) \sin 2\hat{B} = 2(d\hat{B}/d\omega) \cos 2\hat{B}$, so that the modulus of the last term in (51) is smaller than $(2A_{\text{env}})^{\frac{1}{2}}$. Thus, for $A_{\text{env}} = 0.01$, 0.02, or 0.05, the error made by neglecting the second term in (50) is certainly less than ± 14, ± 20, or ± 32 percent, respectively. Usually the error is still smaller because Q_C is somewhat larger than Q_L and because of the approximations in the preceding calculation.

3.2 Low-Pass Filters

Neglecting this second term, for a low-pass filter (50) becomes

$$\delta\hat{A} = \frac{1}{Q_s}\omega\frac{d\hat{B}}{d\omega} = \frac{1}{Q_s}\Omega\frac{d\hat{B}}{d\Omega}. \qquad (52)$$

The fact that $\hat{B}$ and B are practically indistinguishable from each other in the major part

[17] See footnote reference 2, p. 88.

of the transmission range does not imply that $d\hat{B}/d\Omega$ and $dB/d\Omega$ are nearly equal, because the oscillations of $\hat{B}$, though of small amplitude, increase in frequency with the number of filter sections. The following rigorous formula is derived from (23).

$$\frac{d\hat{B}}{d\omega}=e\exp\left[A_{\text{env}}-2\hat{A}\right]\frac{dB}{d\omega}+\frac{1}{2}\frac{dA_{\text{env}}}{d\omega}\sin 2\hat{B}, \quad (53)$$

and shows that $d\hat{B}/d\omega$ and $dB/d\omega$ only coincide at frequencies of perfect matching where $A_{\text{env}}=dA_{\text{env}}/d\omega=\hat{A}=0$. Formula (53) shows that a first difference between $d\hat{B}/d\omega$ and $dB/d\omega$ arises from the factor of $dB/d\omega$, which is approximately

$$e\exp\left[A_{\text{env}}-2\hat{A}\right]=1+A_{\text{env}}-2\hat{A} \\ =1+A_{\text{env}}\cos 2B$$

so that for $A_{\text{env}}<0.05$ neper, this factor does not differ from unity by more than ± 5 percent, which is usually negligible. The second term of (53) is oscillatory and its amplitude $\frac{1}{2}(dA_{\text{env}}/d\omega)$ is usually much smaller than $dB/d\omega$. This is because $dB/d\omega$ starts from a finite value for $\omega=0$ and increases steadily; whereas A_{env}, being small and slowly varying except at frequencies quite close to the cut-off frequency, its derivative is much smaller. The error made by neglecting this second term is hard to estimate; as $dB/d\omega$ is the sum of the contributions of all the sections, whereas $(dA_{\text{env}}/d\omega)$ does not depend on the number of sections, the error will be more important for filters consisting of a small number of sections.

The slope of the image phase of a low-pass filter is given by[18]

$$\frac{dB}{d\Omega}=2(1-\Omega^2)^{-\frac{1}{2}}\sum_{i=1}^{n}\frac{m_i}{1-(1-m_i^2)\Omega^2}. \quad (54)$$

At zero frequency, this reduces to

$$(dB/d\Omega)_o=2\sum m_i$$

and $dB/d\Omega$ monotonically increases with Ω.

At the theoretical cut-off frequency, all the approximations become incorrect and even the rigorous formula (53) becomes indeterminate. A rigorous formula for a symmetrical filter with constant-k image impedances is

$$\left(\frac{dB}{d\Omega}\right)_{\Omega=1}=\frac{2M/\mu}{1+M^2/\mu^2}(\mu^2+M+N-P/M),$$

where

$$M=\sum\frac{1}{m_i},\quad N=\sum\frac{1}{m_im_j},\quad P=\sum\frac{1}{m_im_jm_k}.$$

As the Q of the inductors is zero at zero frequency and starts increasing linearly with frequency, it is convenient to introduce the notation

$$Q_o=Q_L/\Omega, \quad (55)$$

where Q_o is the value of Q_L at the cut-off frequency, if the linear law of increase holds up to the cut-off. Neglecting the dissipation of the capacitors, a practical approximate formula for a low-pass filter is

$$\delta\hat{A}=\frac{1}{2Q_o}\frac{dB}{d\Omega}. \quad (56)$$

3.3 Band-Pass Filters

Formulas (50) and (53) also hold for band-pass filters, but the explicit calculation of $dB/d\omega$ differs from the low-pass-filter case, in which $\omega dB/d\omega$ was equal to $\Omega dB/d\Omega$.

For frequency-symmetrical band-pass filters, the transformation from Ω to ω is given by (7) and $dB/d\omega$ must be calculated from $(dB/d\Omega)(d\Omega/d\omega)$. One finally obtains a practical approximate formula similar to (56)

$$\delta\hat{A}=\frac{1}{Q_s\eta}\frac{dB}{d\Omega}, \quad (57)$$

where $dB/d\Omega$ is still given by (54) and

$$\eta=\frac{1}{\omega}\frac{d\omega}{d\Omega}=\frac{f(f_c-f_{-c})}{f^2+f_o^2}. \quad (58)$$

For a band-pass filter of given Q_L and with negligible dissipation in the capacitors, the factor $1/Q_s\eta=1/2Q_L\eta$ in (51) replaces the factor $1/2Q_o$ of the low-pass expression (56). This shows that the Q at the cut-off frequency of the equivalent low-pass filter is ηQ_L. As for narrow-band-pass filters, η is approximately the half fractional bandwidth

$$\eta=\frac{f_c-f_{-c}}{2f_o},$$

the equivalent Q is considerably reduced.

For dissymmetrical band-pass filters, the calculation of $dB/d\omega$ is simplified by using transformation (14); this gives

$$\omega\frac{dB}{d\omega}=\frac{2\cosh(\chi+\gamma_\infty)\cosh(\chi-\gamma_\infty)}{\sinh 2\gamma_\infty} \\ \times\sum_{i=1}^{n}\operatorname{sech}(\chi-\gamma_i).$$

[18] W. Cauer, "Theorie der Linearen Wechselstromschaltungen," I, Akademische Verlaggesellschaft, Becker und Erler, Leipzig, 1941; p. 324.

At the mid-band frequency this becomes

$$\left(\omega\frac{dB}{d\omega}\right)_{\omega_0} = \coth \gamma_\infty \sum \operatorname{sech} \gamma_i.$$

3.4 Attenuation Poles

Another important effect of the dissipation is to produce a finite attenuation at the attenuation poles. Consider a low-pass filter having an attenuation pole at f_1 (γ_1 on transformed scale). The sections having no poles at f_1 contribute a finite amount to the total image attenuation; the section having a pole at f_1 produces an image attenuation given by

$$A_1 = \log \frac{2Q_s}{e \exp [2\gamma_1] - 1} = \log 2Q_s(\Omega_1^2 - 1),$$

which is proved by E. A. Guillemin.[19] For a symmetrical band-pass filter, the γ scale of the equivalent low-pass filter must be used and Q_s be replaced by ηQ_s.

4. *Appendix*

4.1 Derivation of Formulas for Symmetrical Filters

As any symmetrical filter can be reduced to a lattice structure, the calculations will be based on the lattice normalized impedances $z_1 = jx_1$ and $z_2 = jx_2$.

The usual expression of the effective attenuation and phase is

$$e \exp [\hat{A} + j\hat{B}] = \frac{(1+z_1)(1+z_2)}{(z_2 - z_1)}. \tag{59}$$

Separating real and imaginary parts, this gives

$$e^{\hat{A}} \cos \hat{B} = \frac{x_1 + x_2}{x_2 - x_1}; \quad e^{\hat{A}} \sin \hat{B} = \frac{x_1x_2 - 1}{x_2 - x_1}. \tag{60}$$

Squaring and adding both equations (60) gives

$$e \exp [2\hat{A}] = 1 + \left(\frac{x_1x_2 + 1}{x_2 - x_1}\right)^2 \tag{61}$$

and by division one gets

$$\tan \hat{B} = \frac{x_1x_2 - 1}{x_1 + x_2}. \tag{62}$$

By elimination of x_1 and x_2 between (61), (62) and $w^2 = z_1z_2 = -x_1x_2$ and $\tan^2 B/2 = -x_1/x_2$, formulas (18) and (23) are obtained.

If dissipation is present, each inductive impedance is multiplied by $\alpha = 1 - j/Q_L$ and each capacitive impedance is divided by a factor $\beta = 1 - j/Q_C$. This transforms any purely reactive impedance $Z(j\omega)$ into an impedance

$$Z^*(j\omega) = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} Z[j\omega(\alpha\beta)^{\frac{1}{2}}]. \tag{63}$$

For a pure reactance, and α and β not too different from unity, this gives as a first approximation[20]

$$Z^* = jX + R = jX + \frac{X}{Q_d} + \frac{\omega}{Q_s}\frac{dX}{d\omega}. \tag{64}$$

Writing (59) with the dissipative impedances $z_1^* = jx_1 + r_1 = z_1 + r_1$ and $z_2^* = z_2 + r_2$ and keeping only first-orders terms, one obtains

$$e \exp [\hat{A}^* + j\hat{B}^*] = \frac{(1+z_1)(1+z_2)}{z_2 - z_1} \times \left[1 + \frac{r_1}{1+z_1} + \frac{r_2}{1+z_2} + \frac{r_2 - r_1}{z_2 - z_1}\right]. \tag{65}$$

Dividing by (59), the first member of (65) is approximately

$$e \exp [\delta\hat{A} + j\delta\hat{B}] = 1 + \delta\hat{A} + j\delta\hat{B}$$

and $1 + \delta\hat{A}$ is the real part of the factor between square brackets in (65). The last term being purely imaginary, we have

$$\delta\hat{A} = \operatorname{Re}\left(\frac{r_1}{1+z_1} + \frac{r_2}{1+z_2}\right) = \frac{r_1}{1+x_1^2} + \frac{r_2}{1+x_2^2}. \tag{66}$$

Replacing r_1 and r_2 by their expressions (64), we have

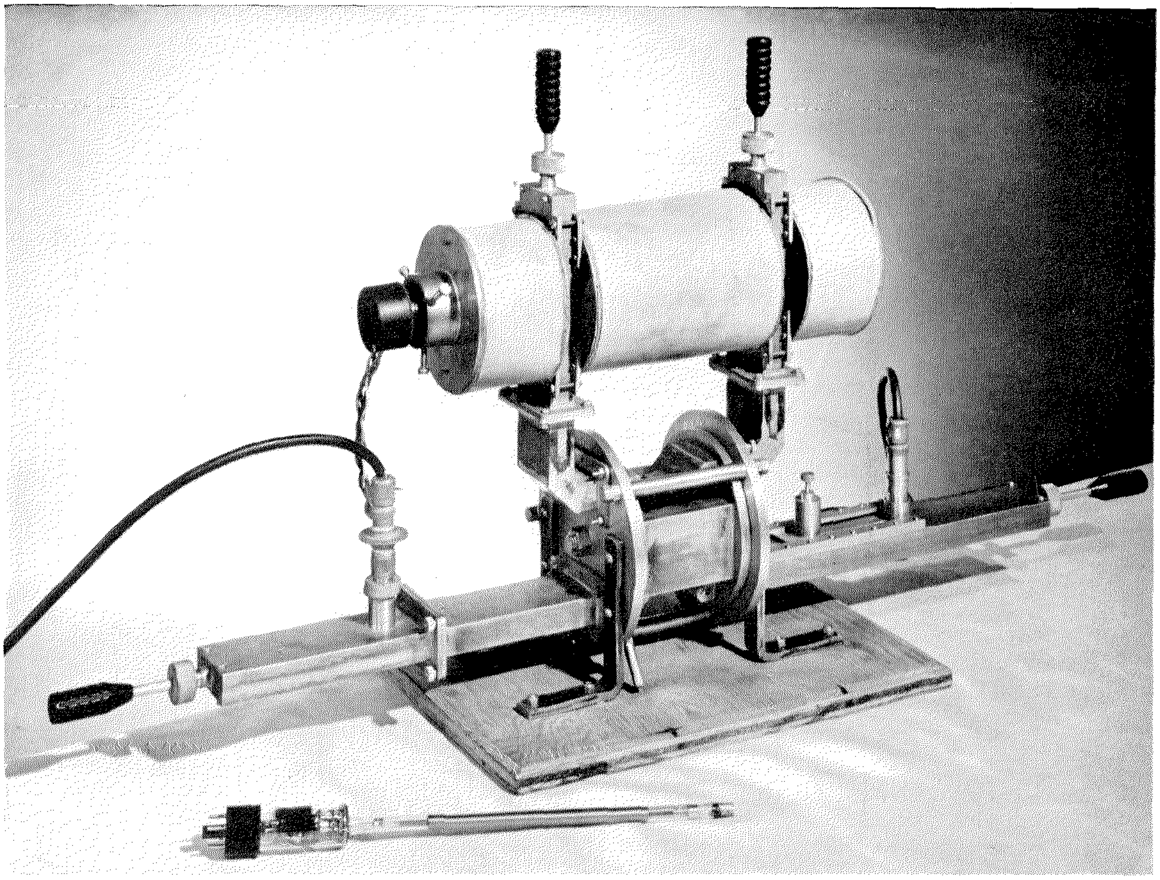
$$\delta\hat{A} + \frac{1}{Q_d}\left(\frac{x_1}{1+x_1^2} + \frac{x_2}{1+x_2^2}\right) + \frac{\omega}{Q_s}\left(\frac{dx_1/d\omega}{1+x_1^2} + \frac{dx_2/d\omega}{1+x_2^2}\right). \tag{67}$$

By derivation of (62), the last factor of (67) is proved to be $dB/d\omega$ and by eliminating x_1x_2 with the help of (60) and (61), (67) is transformed into (50).

[19] E. A. Guillemin, "The Effect of Incidental Dissipation in Filters," *Electronics*, v. 19, pp. 130–135; October, 1946.

[20] Footnote reference 6, p. 445. In this reference, a first-order term corresponding to X/Q_d in (64) is neglected.

Recent Telecommunication Development



TRAVELLING-WAVE AMPLIFIER—An interesting demonstration of a travelling-wave amplifier was given by Standard Telephones and Cables, Limited, at a recent exhibition held in London by the Physical Society. The tube operates at wavelengths in the region of 10 centimeters (3,000 megacycles per second), and is capable of giving amplification of 20 to 30 decibels. The company also makes travelling-wave tubes for the wave band from 6 to 8 centimeters, and the accompanying photograph shows one of these tubes with its associated circuit equipment. The characteristics of these tubes are as follows:

Beam Voltage	1400 volts
Gain	20 to 30 decibels
Maximum Power Output	100 milliwatts
Noise Factor	20 decibels

Measurements on one of these tubes showed the gain to remain constant to within ±1.5 decibels from 6.0 to 8.3 centimeters, a total bandwidth of about 1400 megacycles.

Tubes have also been made in which a noise factor of 13 decibels has been obtained; these, however, are as yet only in the experimental stage.

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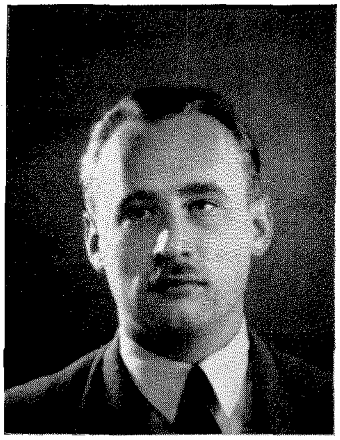
Rotary Automatic Telephone Lines in Service and On Order as of December 31, 1948

Country	Central Office Lines	Private Automatic Branch Exchange Lines	Total Lines
Argentina	——	6,760	6,760
Afghanistan	1,000	——	1,000
Belgium	438,384	81,513	519,897
Bermuda	4,140	80	4,220
Bolivia	500	60	560
Brazil	189,600	10,000	199,600
Chile	——	5,237	5,237
China	79,300	——	79,300
Czechoslovakia	37,800	3,250	41,050
Denmark	201,077	25,285	226,362
Dominican Republic	1,000	180	1,180
Egypt	53,800	——	53,800
Finland	——	400	400
France	567,450	21,500	588,950
Great Britain	20,300	400	20,700
Greece	——	565	565
Honduras	1,120	——	1,120
Hungary	94,090	5,374	99,464
Iceland	——	148	148
Iran	——	170	170
Italy	85,586	26,162	111,748
Jugoslavia	——	2,200	2,200
Luxemburg	3,620	463	4,083
Martinique	2,000	——	2,000
Mexico	44,100	6,070	50,170
Netherlands	125,680	20,505	146,185
Netherlands East Indies	——	1,000	1,000
Netherlands West Indies	1,200	——	1,200
New Zealand	84,330	5,626	89,956
Norway	147,326	20,574	167,900
Peru	30,400	4,900	35,300
Poland	1,300	——	1,300
Porto Rico	10,000	——	10,000
Portugal	——	661	661
Rumania	98,400	9,635	108,035
Spain and Canary Islands	340,860	20,154	361,014
Switzerland	270,836	2,100	272,936
Syria	15,500	——	15,500
Turkey	24,500	726	25,226
Uruguay	——	348	348
United States of America	53,880	6,364	60,244
Venezuela	——	540	540
Others	——	317	317
Total	3,029,079	289,267	3,318,346

Contributors to This Issue



V. BELEVITCH

VITOLD BELEVITCH was born at Helsingfors, Finland, on March 2, 1921. He studied until July, 1936, at the Notre-Dame de la Paix College at Namur. He then followed an engineering course at the University of Louvain and graduated in 1942 with an engineering degree.

Employed by the Bell Telephone Manufacturing Company at Antwerp in October, 1942, in the transmission department, he undertook various studies relating in particular to special transmission problems. In October, 1945, he received a doctor's degree in applied science from the University of Louvain.

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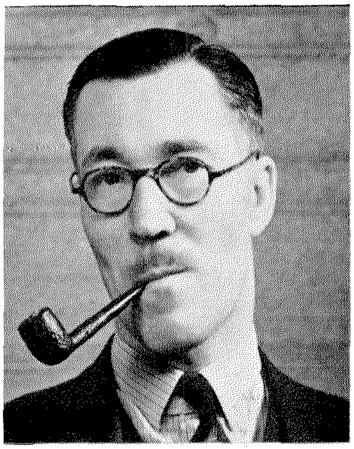

F. H. BRAY

F. H. BRAY was born in Devon, England, in 1906. He attended the Imperial College of Science in London on a Royal Scholarship and obtained a degree in engineering, followed by a year's postgraduate study.

He joined the International Telephone and Telegraph Corporation laboratories, Hendon, in 1928, and was engaged on circuit design and development of automatic telephone systems. Later, he was responsible for the design of factory and installation testing equipment and, since 1937, has been in charge of the switching systems circuit laboratory.

Mr. Bray was elected an Associate Member of the Institution of Electrical Engineers in 1935.

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C. W. EARP was born at Cheltenham, Gloucestershire, England, on July 14, 1905. He received the B.A. degree with First Class Honours in 1927 from Cambridge University.

In September, 1927, he joined the International Standard Electric Corporation at New Southgate, and until 1929 assisted in the study of high-frequency transatlantic transmission.

In 1929, he joined the Paris laboratory as a radio development engineer, where he was chiefly concerned with receiver design, including ship-shore telephone equipments and single-side-band technique.

In 1933, he joined the broadcast receiver development group, which was then transferred from Paris to the Kolster Brandes factory at Sidcup, England, where he became chief of the advance development section.

In 1935, he joined Standard Telephones and Cables at New Southgate as section head for development of radio receivers and direction finders. In 1940, he became head of the newly formed advance development section of the radio division.

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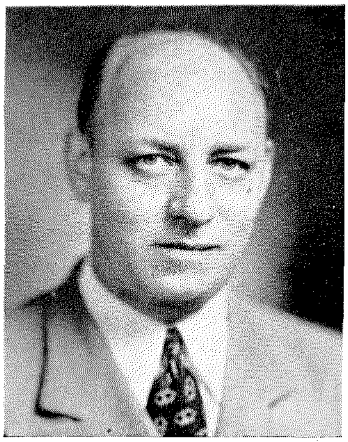
C. W. EARP

PER E. ERIKSON received his degree in electrical engineering from the Royal Institute of Technology in Stockholm in 1903, and joined the Western Electric Company in New York as a shop student the same year.

Assigned to the engineering department, he was engaged on the early development of loading coils and balanced toll cables. Appointed transmission engineer for Europe in 1909 with headquarters in London, he was in charge of the construction of the London-Birmingham cable, the first loaded long-distance cable installed in Europe.

During 1918, he carried out the reconstruction of the Rio de Janeiro–San

PER E. ERIKSON



Sidney Frankel

Paulo toll line, the first of its kind in Brazil to be equipped with repeaters and loaded toll entrance cables. As assistant European chief engineer of the International Western Electric Company, he had charge of the design of toll transmission systems, which were being introduced into various European countries after the first World War. In 1928, he was made assistant vice president of the International Standard Electric Corporation and European chief engineer in 1930.

Mr. Erikson has been associated with the Comité Consultatif International Téléphonique since 1925. From 1929 to date, he has been a delegate for the International Telephone and Telegraph System operating companies at meetings of that body and since 1934, he has been secretary of the System's committee.

He is a Member of the Institution of Electrical Engineers, London, and a Fellow of the American Institute of Electrical Engineers.

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Sidney Frankel was born on October 6, 1910 in New York City. Rensselaer Polytechnic Institute conferred three degrees on him: the B.A. degree in electrical engineering in 1931, and in mathematics the M.A. degree in 1934 and the Ph.D. degree in 1936. He was an instructor in mathematics from 1931 to 1933.

For a year after leaving college, Dr. Frankel served as a sound-recording engineer with the Brooklyn Vitaphone Corporation. In 1937–1938, he was an assistant engineer in the design and development of electronic flight instruments for the Eclipse Aviation Corporation.

He joined the Federal Telegraph Company staff at Newark, New Jersey, in 1938 as an engineer on the design and development of radio transmitters. In 1943, he was transferred to Federal Telephone and Radio Laboratories, now Federal Telecommunication Laboratories. At present he is engaged in the development of components for microwave systems.

He is a member of Sigma Xi and a Senior Member of the Institute of Radio Engineers.

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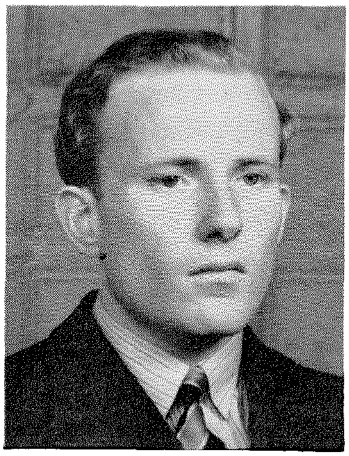

Ronald M. Godfrey

Ronald M. Godfrey was born at Wood Green, Middlesex, England on April 22, 1923. After studying at the City and Guilds College, he received the degree of B.Sc. (Eng.) Hons. at the University of London in 1943.

On graduation, he joined the radio division of Standard Telephones and Cables, where, as an engineer in the advanced development department, he assisted in the design of specialised electronic communication and direction-finding equipment. Recently, as head of a development group, he has been concerned with the problems of high-frequency direction finding.

Mr. Godfrey is a Graduate Member of the Institution of Electrical Engineers.

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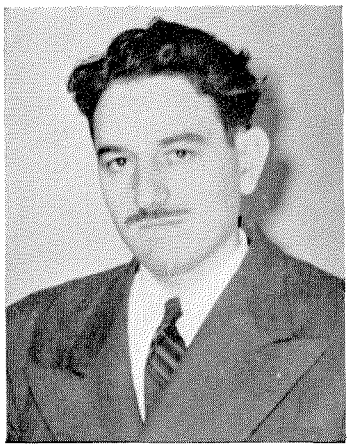
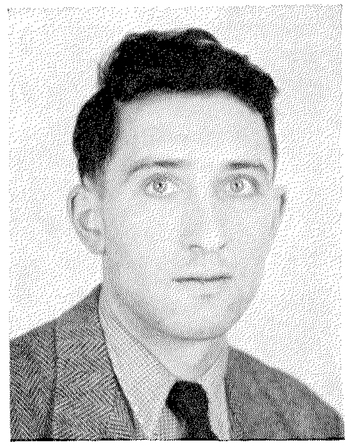


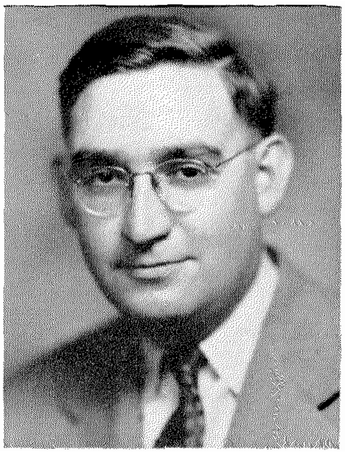
D. D. Grieg

D. D. Grieg was born on February 26, 1915 in London, England. He received his early schooling in England and the B.S. degree in electrical engineering from the College of the City of New York.

From 1936 to 1940, he was in charge of the television department of the Davega Radio Company. In early 1941, he taught radio communication in the Brooklyn Technical High School. Since 1941, he has been a research engineer for Federal Telecommunication Laboratories. He is now a division head and has charge of the television and communication departments.



Salvatore Milazzo



SIDNEY MOSKOWITZ

Mr. Grieg is a Senior Member of the Institute of Radio Engineers and a Member of the American Institute of Electrical Engineers. He has served on several technical committees including the Television Committee of the Radio Technical Planning Board and those on Television Relays and Studio-Transmitter Links of the Radio Manufacturers Association. He is the author of several technical papers and holds many patents in the field of radio.

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SALVATORE MILAZZO was born in New York City in 1917. He received the B.A. degree from Brooklyn College in 1943. He has been with Federal Telecommunication Laboratories since 1943 and is engaged in the development of microwave antennas.

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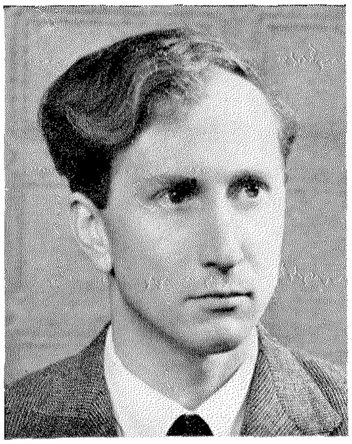

D. S. RIDLER

SIDNEY MOSKOWITZ was born in Brooklyn, N.Y., on February 23, 1919. In 1940, he received the B.E.E. degree from the College of the City of New York. From 1941 to 1945 he was a member of the evening-session staff at that College.

While teaching evenings, he was also engaged in the development of electronic apparatus for the Industrial Scientific Corporation in New York. In 1943 Mr. Moskowitz joined the Federal Telecommunication Laboratories, where he has been active in the design and development of pulse-time-modulation systems.

Mr. Moskowitz is a Member of the Institute of Radio Engineers and a member of Tau Beta Pi.

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D. S. RIDLER was born in London in 1921. He joined Standard Telephones and Cables in 1939, and completed a 3-year apprenticeship course as a laboratory assistant, obtaining the Higher National Certificate with an endorsement giving exemption from the entrance examination of the Institution of Electrical Engineers. Mr. Ridler has been actively engaged in laboratory work and, more particularly, with the application of gas-filled tubes. He is now located at Standard Telecommunication Laboratories at Enfield.

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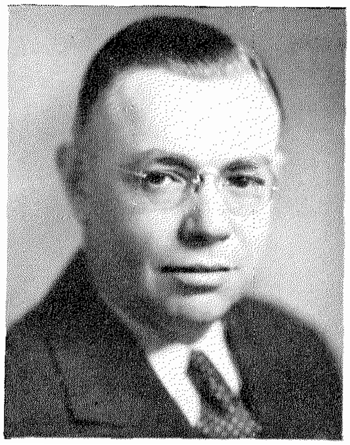
PETER C. SANDRETTO was born in Pont Canavese, Italy, on April 14, 1907. He received the BSEE and EE degrees from Purdue University in 1930 and 1938. He also did graduate work at Northwestern University and was graduated from the Air Staff Course of the Command and General Staff School at Fort Leavenworth in 1946.

From 1925 to 1930, he was a radio operator and engineer for several radio broadcasting stations. During the following two years, he was with Bell Telephone Laboratories. In 1932, he joined United Air Lines and served as superintendent of their communications laboratories until 1942, when he entered the U. S. Air Forces with the rank of major, advancing later to colonel.

He joined the I. T. & T. System in 1946 and is at present serving as assistant technical director of Federal Telecommunication Laboratories.



PETER C. SANDRETTO

During the war, he was decorated with the Bronze Star for his achievements in the Central Pacific. He is a member of Eta Kappa Nu, honorary electrical engineering fraternity, a Senior Member of the Institute of Radio Engineers, Member of the Institute of Navigation, an Associate Member of the Institution of Electrical Engineers, and a Member of the Institute of the Aeronautical Sciences. He has served on various committees of the Radio Technical Commission for Aeronautics ever since its inception in 1935. He is the author of the text book "Principles of Aeronautical Radio Engineering."

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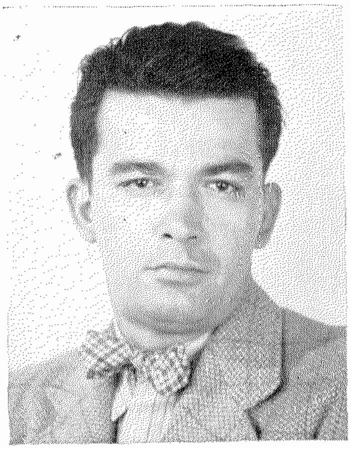
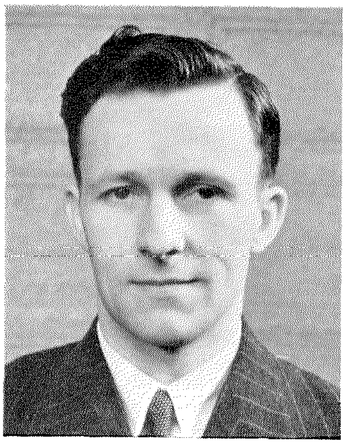


WILLIAM SICHAK



W. A. G. WALSH

WILLIAM SICHAK was born on January 7, 1916, at Lyndora, Pennsylvania. He received the B.A. degree in physics from Allegheny College in 1942.

From May, 1942, to November, 1945, he was engaged in developing microwave radar antennas at the Radiation Laboratory, Massachusetts Institute of Technology. Since November, 1945, he has been with Federal Telecommunication Laboratories, working on microwave antennas and allied subjects.

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W. A. G. WALSH was born in London in 1923. He became a member of the laboratory staff of Standard Telephones and Cables in 1939.

He was on active duty with the Royal Navy from 1941 until 1946. Since his return to the company, he has been actively engaged in the application of gas-filled tubes to switching problems. Mr. Walsh is taking part-time courses at Northampton Polytechnic for his engineering diploma.

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A. J. WARNER

A. J. Warner was born in London, England, on March 25, 1913. He joined the staff of Standard Telephones and Cables in 1930 as a metallurgical chemist and in 1932 was granted a leave of absence. In 1935, he received the B.Sc. degree in chemistry from the University of London and then returned as a chemist on raw materials and on research problems related to electrical insulation.

In 1940, Mr. Warner was transferred to the new insulants factory at Enfield, Middlesex, England, and in 1941, to the International Telephone and Radio Manufacturing Corporation in Newark, New Jersey, U. S. A. Later this organization became part of Federal Telephone and Radio Corporation. Near the end of 1945, he was named manager of the dielectrics laboratories, of Federal Telecommunication Laboratories.

Mr. Warner has been designated by the American Society for Testing Materials as one of six members of the Advisory Group that will assist the Armed Forces in technical problems arising in the research program on plastics recently initiated at Princeton University.

He maintains memberships in three professional societies in Great Britain: Chemical Society, Royal Institute of Chemistry, and Society of Chemical Industry. In the U. S. A., he is a member of the American Chemical Society, of Committees D-9 and D-20 of the American Society for Testing Materials, President of the Newark Chapter of the Society of Plastics Engineers, Vice Chairman of the Technical Committee of the Society of the Plastics Industry, and a member of the American Institute of Electrical Engineers.

INTERNATIONAL TELEPHONE AND TELEGRAPH CORPORATION

Associate Manufacturing and Sales Companies

United States of America

International Standard Electric Corporation, New York, New York

Federal Telephone and Radio Corporation, Clifton, New Jersey

International Standard Trading Corporation, New York, New York

Great Britain and Dominions

Standard Telephones and Cables, Limited, London, England

Branch Offices: Birmingham, Bristol, Leeds, Manchester, England; Glasgow, Scotland; Dublin, Ireland; Cairo, Egypt; Calcutta, India; Johannesburg, South Africa

Creed and Company, Limited, Croydon, England

International Marine Radio Company Limited, Liverpool, England

Kolster-Brandes Limited, Sidcup, England

Standard Telephones and Cables Pty. Limited, Sydney, Australia

Branch Offices: Melbourne, Australia; Wellington, New Zealand

Silovac Ele trical Products Pty. Limited, Sydney, Australia

Austral Standard Cables Pty. Limited, Melbourne, Australia

New Zealand Electric Totalisators Limited, Wellington, New Zealand

Federal E ectric Manufacturing Company, Ltd., Montr al, Canada

South America

Compañía Standard Electric Argentina, Sociedad Anónima, Industrial y Comercial, Buenos Aires, Argentina

Standard Electrica, S.A., Rio de Janeiro, Brazil

Compañía Standard E ectric, S.A.C., Santiago, Chile

Europe and Far East

Vereinigte Telephon- und Telegraphenfabriks Aktien-Gesellschaft Czeija, Nissl and Company, Vienna, Austria

Bell Telephone Manufacturing Company, A twerp, Belgium

China Electric Company, Limited, Shanghai, China

Standard Elec ric Aktieselskab, Copenhagen, Denmark

Compagnie Générale de Constructions Téléphoniques, Paris, France

Le Matériel Téléphonique, Paris, France

Les Téléimprim urs, Paris, France

Lignes Télégraphiques et Téléphoniques, Paris, France

Ferdinand Schuchhardt Be liner Fernsprech- und Telegraphenwerk Aktiengesellschaft, Berlin, Germany

C. Lorenz, A.G. and Subsidiaries, B lin, Germany

Mix & Genest Aktienges llschaft and Subsidiaries, Berlin, Germany

Süddeutsche Apparatefabrik Gesellschaft M.B.H., Nuremberg, Germany

Telephonfabrik Berliner A.G. and Subsidiaries, Be in, Germany

Neder andsche Standard Electric Maatschappij N.V., Hague, Netherlands

Dial Telefonkereskedelmi Részvény Társaság, Budapest, Hungary

Standard Villamossági Részvény Társaság, Budapest, Hungary

Telefongyár R.T., Budapest, Hungary

Fabbrica Apparecchiature per Comunicazioni Elettriche, Milan, Italy

Standard Elettrica Italiana, Milan, Italy

Societa Italiana Reti Telefoniche Inte urbane, Milan, Italy

Nippon Electric Compan , Limited, Tokyo, Japan

Sumitomo Electric Industries, Limited, Osaka, Japan

Standard Telefon- og Kabelfabrik A/S, Oslo, Norway

Standard Electrica, Lisbon, Portugal

Compañía Radio Aerea Maritima Española, Madrid, Spain

Standard Eléctrica, S.A., Madrid, Spain

Aktiebolaget Standard Radiofabrik, Stockholm, Sweden

Standard Telephone et Radio S.A., Zurich, Switzerland

Telephone Operating Systems

Compañía Telefónica Argentina, Buenos Aires, Argentina

Compañía Telegráfico-Telefónica Comercial, Bu nos Aires, Argentina

Compañía Telegráfico-Telefónica del Pla a, Buenos Aires, Argentina

Companhia Telefonica Paranaense S.A., Curitiba, Brazil

Companhia Telefonica Rio Grandense, Porto Alegre, Brazil

Compañía de Teléfonos de Chile, Santiago, Chile

Compañia Telefónica de Magallanes S.A., Punta Arenas, Chile

Cuban Telephone Company, Havana, Cuba

Cuban American Telephone and Telegraph Company, Havana, Cuba

Mexican Telephone and Telegraph Company, Mexico City, Mexico

Compañía Peruana de Teléfonos Limitada, Lima, Peru

Porto Rico Telephone Company, San Juan, Puerto Rico

Shanghai Telephone Company, Federal, Inc., U.S.A., Shanghai, China

Radiotelephone and Radiotelegraph Operating Companies

Compañía Internacional de Radio, Buenos Aires, Argentina

Compañía Internacional de Radio Boliviana, La Paz, Bolivia

Companhia Radio In ernacional do Brasil, Rio de Janeiro, Brazil

Compañía Internacional de Radio, S.A., Santiago, Chile

Radio Corporation of Cuba, Havana, Cuba

Radio Corporation of Porto Rico, Santurce, Puerto Rico[1]

[1] Radiotelephone and Radio Broadcasting services.

Cable and Radiotelegraph Operating Compani

(Controlled by American Cable & Radio Corporation)

The Commercial Cable Company, New York, New York[2]

Mackay Radio and Telegraph Company, New York, New York[3]

All America Cables and Radio, Inc., New York, New York[4]

Sociedad Anónima Radio Argentina, Buenos Aires, Argentina[5]

[2] Cable service. [3] International and Marine Radiotelegraph services.
[4] Cable and Radiotelegraph services. [5] Radiotelegraph service.

Laboratories

Federal Telecommunication Laboratories, Inc., Nutley, New Jersey

Standard Telecommunication Laboratories Ltd., London, England

Laboratoire Central de Télécommunications, Paris, France